4/8/20: Topic 1 SAS 1

4/9/20: Topic 1 SAS 2

4/10/20: Topic 2 SAS 1

4/13/20: Topic 2 SAS 2

4/14/20: Topic 2 SAS 3

4/15/20: Topic 4 SAS 2

4/16/20: Topic 4 SAS 3

4/17/20: Topic 9 SAS 2

4/20/20: Topic 9 SAS 3

4/21/20: Topic 9 SAS 4

4/22/20: Topic 10 SAS 1

4/23/20: Topic 10 SAS 2

4/24/20: Topic 10 SAS 3

Support for students, parents, and guardians:

- Teachers will be available to answer questions through Zoom on the following dates. To access the support call, following the directions below
  - **April 14th, 10:00 a.m. – 10:45 a.m.**
    - Click on the link https://zoom.us/j/891192096, OR
    - Open Zoom app and enter Meeting ID: 891 192 096
  - **April 21st, 10:00 a.m. – 10:45 a.m.**
    - Click on the link https://zoom.us/j/3791568353, OR
    - Open Zoom , click join, and enter Meeting ID: 379 156 8353

Student:____________________________________________Class:_________________Date_____________

## Using inductive reasoning and conjectures

Topic 1 Student Activity Sheet 1; *Overview*



1. **REVIEW** Look at the patterns below. Can you find the next two items in each list and state the rule for finding them?

   a. 2, 4, 6, 8, …

   b. 2, 3, 5, 9, 17, …

   c. 1, -2, 3, -4, 5, …

   d. 1, 4, 9, 16, …

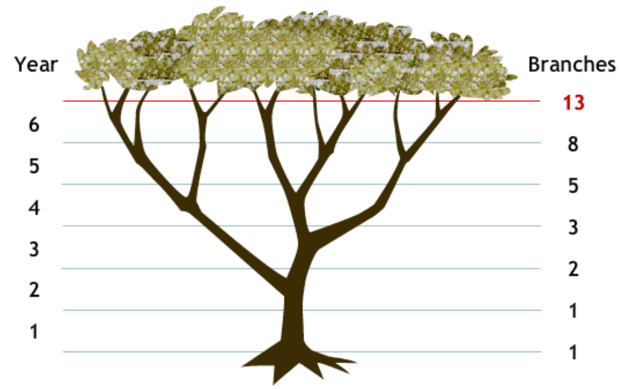


Student:____________________________________________Class:_________________Date_____________

Using inductive reasoning and conjectures

Topic 1 Student Activity Sheet 1; *Overview*






2. In the diagram, the numbers of branches that appear as the tree grows model the ***Fibonacci numbers***. This sequence of numbers is named after the Italian mathematician Leonardo Fibonacci (1170–1250 AD). Can you find the pattern in the number of branches as the tree grows?

3. Fill in the next few numbers in the sequence of Fibonacci numbers.

   1, 1, 2, 3, 5, 8, 13,

4. In the picture below, find as many geometric objects as you can. Make a list of all the geometric objects that you find.



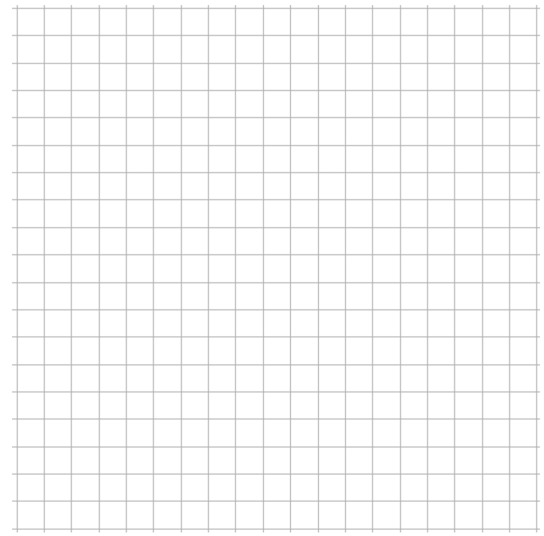




## Using inductive reasoning and conjectures
Topic 1 Student Activity Sheet 1; *Overview*


5. **REVIEW** Graph and label the following points on the coordinate plane.

**A** (1,4)
**B** (-5,0)
**C** (0,8)
**D** (3,-5)
**E** (0,-2)
**F** (-8,-4)
**G** (4,0)
**H** (-7,7)

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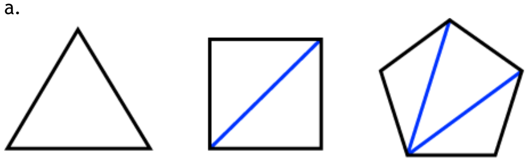
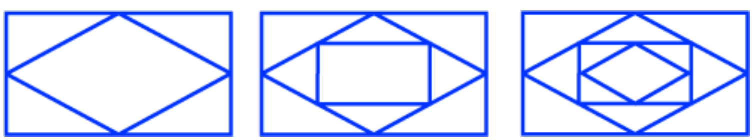
6. **REINFORCE** Draw the next image for each of these patterns. Write a description of the rule represented by the image and explain how to use the rule to find the next figure.

a.



b.



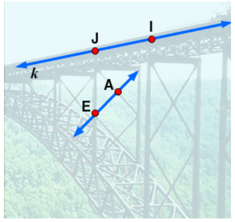


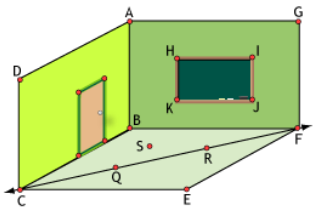


1. **REINFORCE** Find a geometric representation for the following sequence of numbers.

3, 4, 5, 6, 7, …

2. What are two names for the line containing points **A** and **E**?






3. **REINFORCE**

   a. Name two points in the room diagram that are collinear with points **C** and **F**.

   b. Point **J** is noncollinear with points **H** and **K.** Name another point that is noncollinear with points **H** and **K.**

   c. Points **C, Q,** and **S** are coplanar points. Name another point on the floor that is coplanar with **C** and **Q**.

   d. Points **A, B,** and **F** are noncoplanar with point **C**. Name another point in the room that is noncoplanar with **A, B,** and **F**.





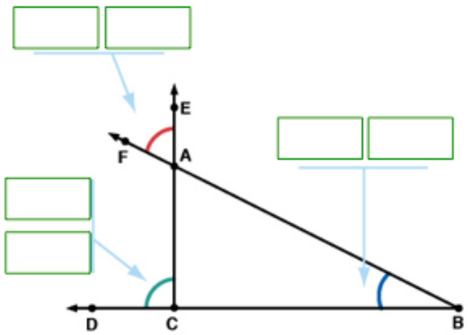
4. Using the notations provided, complete the table by writing in the correct notation for each name and figure.

| | | | |
|---|---|---|---|
| AB | BA | $\overline{AB}$ | $\overline{BA}$ |
| $\overrightarrow{AB}$ | $\overrightarrow{BA}$ | $\overleftrightarrow{AB}$ | $\overleftrightarrow{BA}$ |

| Figure | Name | Notation |
|---|---|---|
|  | Line AB | ☐ or ☐ |
|  | Ray AB | ☐ |
|  | Ray BA | ☐ |
|  | Segment AB | ☐ or ☐ |
|  | The distance between A and B | ☐ or ☐ |

5. Using the angle names provided, label the angles in the diagram below.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ∠B | ∠EAF | ∠A | ∠ECD | ∠FEA | ∠FAE | ∠CBA | ∠DCA |

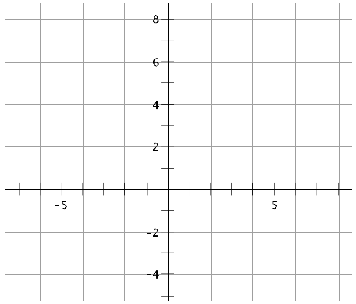



6. Write a definition of supplementary angles. Give an example of two supplementary angles.

7. Write a definition of complementary angles. Give an example of two complementary angles.





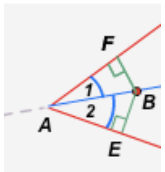



8. **REINFORCE** Suppose ∠**A** and ∠**B** are complementary angles, m∠**A** = $(3x + 5)^\circ$, and m∠**B** = $(2x - 15)^\circ$. Solve for $x$ and then find m∠**A** and m∠**B**.

9. **REINFORCE** The measure of the supplement of an angle is 12 more than twice the measure of the angle. Find the measures of the angle and its supplement.

10. Write a definition for ***angle bisector***, and then sketch an example.

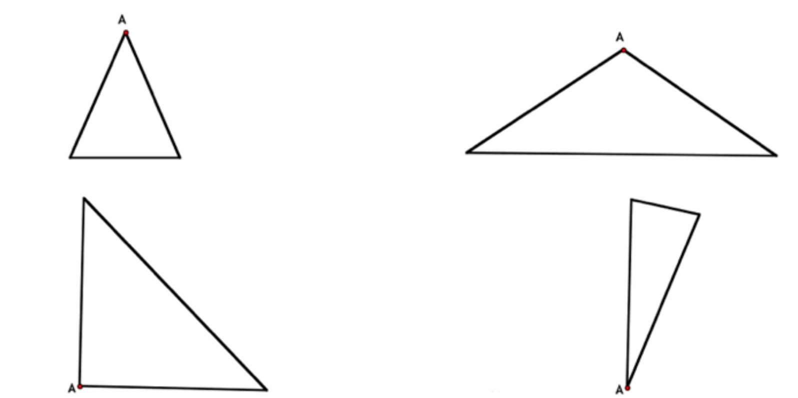
11. **REINFORCE** In the diagram, $\overrightarrow{AB}$ bisects $\angle$**FAE**. **BF** = 5***x*** and **BE** = $x^2$ + 6. Solve for ***x***.



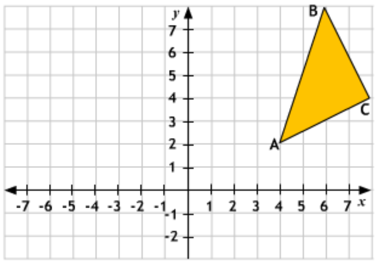





12. **REINFORCE** Below are several isosceles triangles. Construct the angle bisector of $\angle$**A** on each triangle. Then write a conjecture about the angle bisector of the angle formed by the two congruent sides of an isosceles triangle.



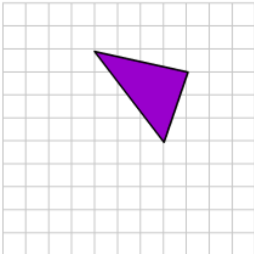
1. Reflect Δ**ABC** across the ***y***-axis.



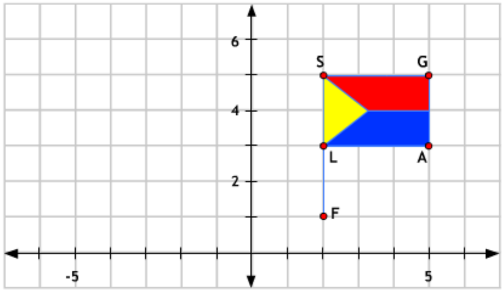


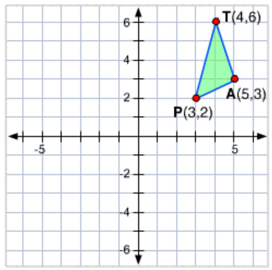



2. Translate the triangle down 5 units.



3. Rotate **FLAGS** 90˚ counterclockwise about the origin.






1. Using Patty Paper, reflect $\triangle$**PAT** across the ***y***-axis. Label the image $\Delta$**P'A'T'.** What are the coordinates of the vertices of $\Delta$**P'A'T'**?

2. Write a conjecture about what happens to the coordinates of a point when you reflect it across the ***y***-axis.

3. Write a conjecture about what happens to the coordinates of a point when you reflect it across the ***x***-axis.

4. Using Patty Paper, reflect $\triangle$**PAT** across the ***x***-axis. Label the image $\triangle$**P''A''T''**. What are the coordinates of the vertices of $\triangle$**P''A''T''**?





| ***x*** | ***y*** | ***-x*** | ***-y*** |
|---|---|---|---|

5. Use the answer choices shown above to complete the following statements.

   a. A reflection across the ***x***-axis maps the point (***x***,***y***) to the point (______ , ______).

   b. A reflection across the ***y***-axis maps the point (***x***,***y***) to the point (______ , ______).

6. Using ordered pair rule notation, rewrite the rules you completed in question 5.

7. Compare the coordinates of **C** and **C'**, **O** and **O'**, and **T** and **T'**. Notice what is true about the coordinates of each pre-image (***x***,***y***) and its image (***x***',***y***'). Then use the given answer choices to complete the following statements.

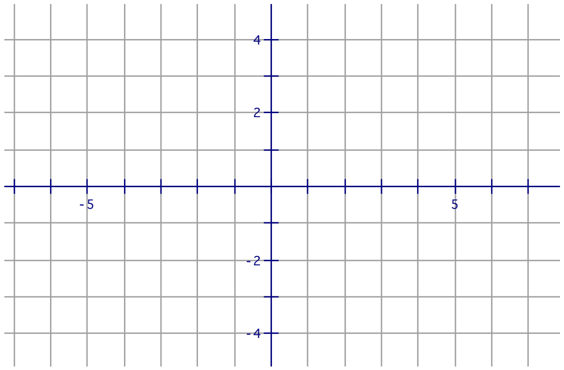
| | | | |
|---|---|---|---|
| ***x*** | ***y*** | **stay the same** | **have their signs changed** |
| **-*x*** | **-*y*** | **are interchanged** | |

   a. When the pre-image (***x***,***y***) is reflected across the line ***y*** = ***x***, the ***x***- and ***y***-coordinates of the pre-image and image ________________________________.

   b. The ordered pair rule for a reflection across the line ***y*** = ***x*** is (***x***,***y***) → (______ , ______).






8. **REINFORCE** Quadrilateral **CDEF** has the following vertices: **C**(1,2), **D**(5,3), **E**(5,1), and **F**(3,-2).

   a. Plot quadrilateral **CDEF** on the grid.

   b. Reflect quadrilateral **CDEF** across the ***x***-axis. What are the coordinates of the image?

   c. Reflect quadrilateral **CDEF** across the ***y***-axis. What are the coordinates of the image?

   d. Reflect quadrilateral **CDEF** across the line ***y*** = ***x***. What are the coordinates of the image?

9. Using the answer choices provided, name the transformation that goes with each ordered pair rule. Assume $a \neq b$.

| | | |
|---|---|---|
| **reflection across $y = x$** | **reflection across the $x$-axis** | **reflection across the $y$-axis** |
| **rotation of 180˚ about (0,0)** | **doesn't match a given transformation** | |

a. $(a,b) \rightarrow (b,a)$ ________________________________________

b. $(a,b) \rightarrow (-a,b)$ ________________________________________

c. $(a,b) \rightarrow (a,-b)$ ________________________________________

d. $(a,b) \rightarrow (-a,-b)$ ________________________________________

e. $(a,b) \rightarrow (-b,-a)$ ________________________________________

10. **REINFORCE** Find the image of the point (5,8) for each transformation described.

a. Reflection across the $x$-axis.

b. Reflection across the $y$-axis.





c. Reflection across the line $y = x$.

d. Reflection across the $x$-axis, followed by a reflection across the $y$-axis.

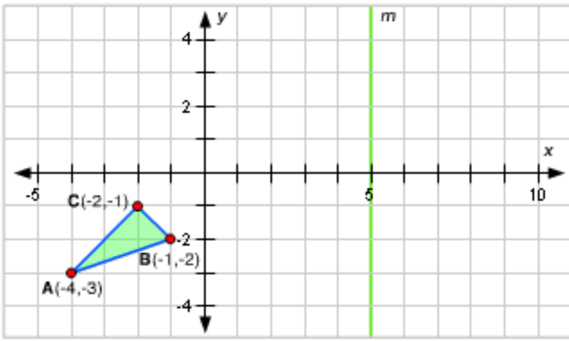
e. Reflection across the $y$-axis, followed by a reflection across the $x$-axis.

f. Rotation about the origin by 180˚.

Student:_______________________________________________Class:_________________Date_____________



1. Use this grid to complete the Patty Paper exercise below.




- Overlay your Patty Paper to copy the ***x***- and ***y***-axes, line ***m***, and **△ABC**.
- Reflect **△ABC** across the ***y***-axis. Mark the locations of **A'**, **B'**, and **C'**.
- Overlay your Patty Paper on the grid to find the coordinates of **A'**, **B'**, and **C'**.
- Reflect **△A'B'C'** across the line ***x*** = ***m***. Mark the locations of **A"**, **B"**, and **C"**.
- Overlay your Patty Paper on the grid to find the coordinates of **A"**, **B"**, and **C"**.

Sketch the results from your Patty Paper on the grid, or attach your Patty Paper to this Student Activity Sheet.



Student:_______________________________________________Class:_________________Date_____________

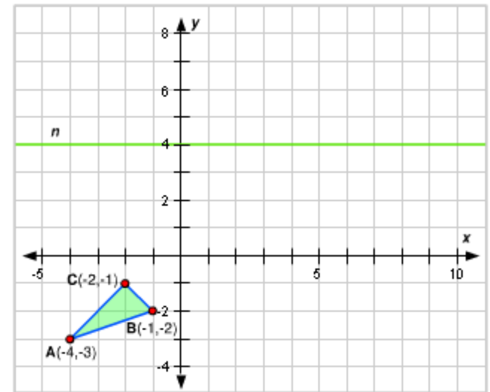


Complete the steps below to investigate what happens when **△A"B"C"** translates vertically.

**Step 1:** Draw **△A"B"C"** and the line ***y*** = 4 on the coordinate grid. Label the line ***n***.

**Step 2:** Draw the reflection image of **△A"B"C"** across the ***x***-axis. Label the reflection image of **△A"B"C"** and record the coordinates of the vertices **D**, **E**, and **F**.

**Step 3:** Now reflect **△DEF** across line ***n*** to get the translation image of **△A"B"C"**. Label the translation image **△D'E'F'** and record the coordinates of the vertices.

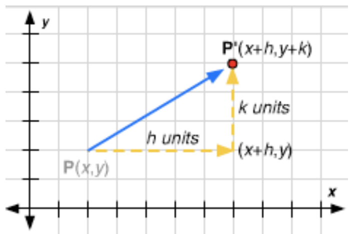



2. What is the ordered pair rule for reflecting **A"** twice to **D'**?

3. What are the single ordered pair rules for translating points **A"**, **B"**, and **C"**?

4. To compare the beginning position of **△ABC** with the ending position of **△D'E'F'**, write single ordered pair rules for corresponding vertices. Then describe the translation in words.

5. Use vector notation to describe the vector on the graph.




6. Using the answer choices provided, complete the following statements.

| | | | |
|---|---|---|---|
| $k < 0$ | $h > 0$ | $x + h$ | $x - h$ |
| $k > 0$ | $h < 0$ | $y + k$ | $y - k$ |

a. If P($x$,$y$) is translated $|h|$ units to the right, the signed value of $h$ is __________.

b. If P($x$,$y$) is translated $|h|$ units to the left, the signed value of $h$ is __________.

c. If P($x$,$y$) is translated $|k|$ units up, the signed value of $k$ is __________.

d. If P($x$,$y$) is translated $|k|$ units down, the signed value of $k$ is __________.

e. A single ordered pair rule for translating P($x$,$y$) $h$ units horizontally and $k$ units vertically is P($x$,$y$) $\xrightarrow{\langle h,k \rangle}$ P' (____________ , ____________).

7. **REINFORCE** A point has coordinates ($x$,$y$). Write an ordered pair rule for a translation that moves the point 5 units to the right and 3 units down.

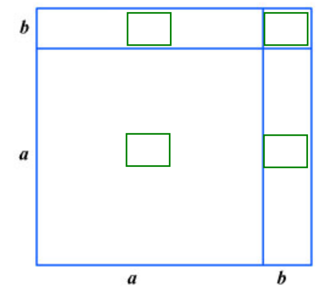




| | | |
|---|---|---|
| $b^2$ | $ab$ | $a^2$ |

1. Use the answer choices above to fill in the blanks in the diagram and show why the mathematical statement is true.

$$(a + b)(a + b) = a^2 + 2ab + b^2$$




2. Solve the following equation. As you complete each step in the solution, take time to think about why that step is true. Show all of your work.

$$4x + 2 = 10$$

## Deductive reasoning, logic, and proof



**Before you continue with the Student Activity Sheet, take some time to review the properties of real numbers shown in the table below.**

**Properties of Equality**

For any real numbers ***a***, ***b***, and ***c***:

| Addition Property | Substitution Property |
|---|---|
| If $a = b$, then $a + c = b + c$. | If $a = b$, then $b$ can substitute for $a$ in any equation. |
| **Subtraction Property** | **Reflexive Property** |
| If $a = b$, then $a - c = b - c$. | $a = a$ |
| **Multiplication Property** | **Symmetric Property** |
| If $a = b$, then $ac = bc$. | If $a = b$, then $b = a$. |
| **Division Property** | **Transitive Property** |
| If $a = b$, then $\frac{a}{c} = \frac{b}{c}$; $c \neq 0$. | If $a = b$, and $b = c$, then $a = c$. |

3. When the properties are listed alongside an equation so that each step is justified, the result is an algebraic proof. Use the properties of equality to complete the justification of the solution to the equation you solved in question 2.

| Statements | Reasons |
|---|---|
| $4x + 2 = 10$ | Given equation |
| $4x + 2 - 2 = 10 - 2$ | ______________________________ |
| $4x = 8$ | Simplify |
| $\frac{4x}{4} = \frac{8}{4}$ | ______________________________ |
| $x = 2$ | Simplify |

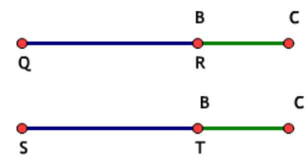




## Deductive reasoning, logic, and proof

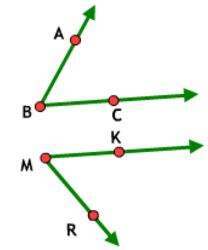


4. Use a property of equality to justify each of the following statements.




If **QR** = **ST**, then **QR** + **BC** = **ST** + **BC**.

a. ______________________________

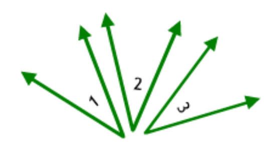



If m∠**ABC** = m∠**KMR**, then m∠**KMR** = m∠**ABC**.

b. ______________________________

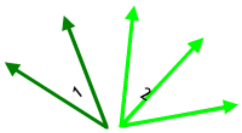
**AX** = **AX**

c. ______________________________



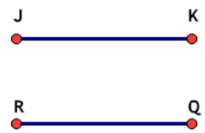

If m∠1 = m∠2 and m∠2 = m∠3, then m∠1 = m∠3.

d. ______________________________



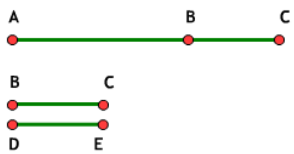

If $2 \cdot m\angle 1 = m\angle 2$,
then $m\angle 1 = \frac{m\angle 2}{2}$.

e. ______________________________



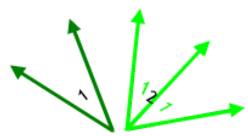

If **JK** + **AB** = **RQ** + **AB**,
then**JK** = **RQ**.

f. ______________________________




From the diagram, **AB** + **BC** = **AC**.
If **BC** = **DE**, then **AB** + **DE** = **AC**.

g. ______________________________



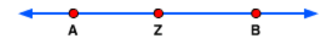

If $m\angle 1 = \frac{1}{2} \cdot m\angle 2$,
then $2 \cdot m\angle 1 = m\angle 2$.

h. ______________________________

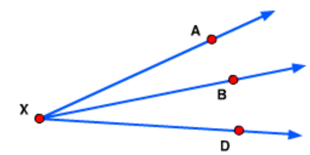




1. Points **A**, **Z**, and **B** lie on $\overline{AB}$. If **AZ** = 2 centimeters and **ZB** = 3 centimeters, what is **AB**? Explain how you found your answer.




2. Now consider a similar question involving angles. Given the angles shown in the diagram, if $m\angle AXB = 15°$ and $m\angle BXD = 20°$, what is $m\angle AXD$? On what assumptions are you basing your answer?



3. Use the given answer choices to complete the statements.

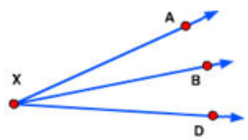
| | | | | | |
|---|---|---|---|---|---|
| **ZB** | m∠**BXD** | m∠**AXD** | **AB** | m∠**AXB** | **AZ** |

**Segment Addition Postulate:**
If **Z** is between **A** and **B**, then

_________ + _________ = _________.


**Angle Addition Postulate:**
If **B** is in the interior of ∠**AXD**, then

_________ + _________ = _________.



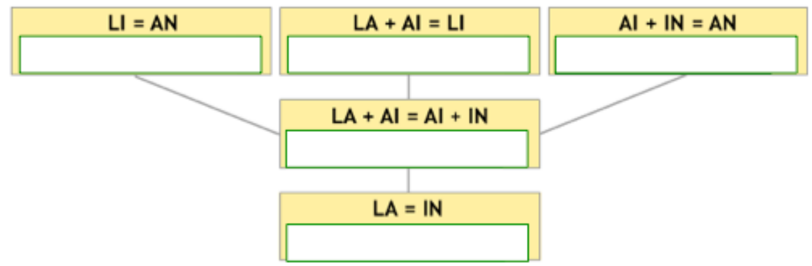





5. Using the answer choices provided, fill in the correct reasons for each of the statements in this flow-chart proof.

| | | | |
|---|---|---|---|
| **Substitution Property** | **Division Property** | **Angle Addition Property** | **Multiplication Property** |
| **Subtraction Property** | **Segment Addition Postulate** | **Addition Property** | **Given** |

Given: **LI** = **AN**
Prove: **LA** = **IN**

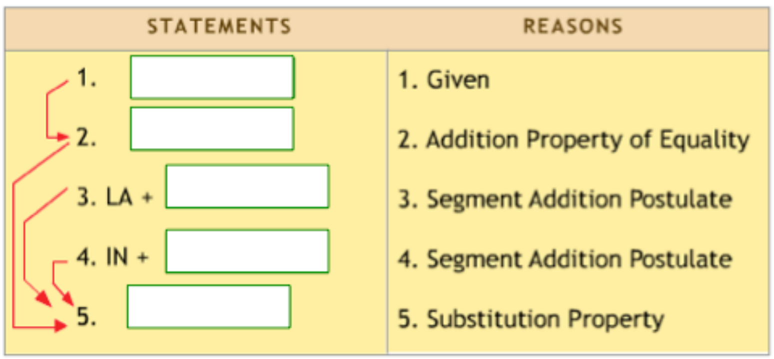
## Deductive reasoning, logic, and proof



6. Using the answer choices provided, fill in the correct reasons for each of the statements in this two-column proof.

| AI = AN | LI = AN | LA = IN | AI = LI | LA + AI = IN + AI |
|---|---|---|---|---|

Given: LA = IN
Prove: LI = AN


7. Using the answer choices provided, fill in both statements and reasons in the flow-chart proof.

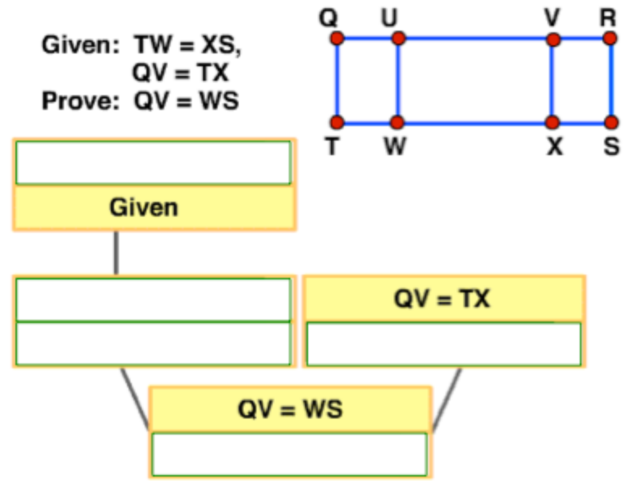
| | | |
|---|---|---|
| Transitive Property | Given | TX = WS |
| TW = XS | Common Segment Theorem | Segment Addition Postulate |





## Deductive reasoning, logic, and proof

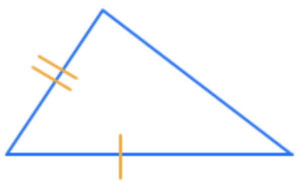




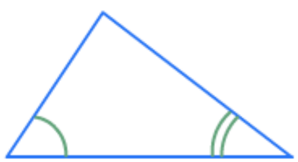
## Congruent triangle postulates



On the diagram, two sides are marked. Mark the angle that is considered the **included angle** in relation to these two sides.



1. On the diagram, two angles are marked. Mark the side that is considered the **included side** in relation to these two angles.



2. Put an "X" in the table to indicate which combinations of three pairs of congruent parts guarantee two triangles congruent.

| | Proves congruence | Does not prove congruence |
|---|---|---|
| **SSS** | | |
| **AAA** | | |
| **SAS** | | |
| **SSA** | | |
| **ASA** | | |
| **SAA** | | |





Congruent triangle postulates



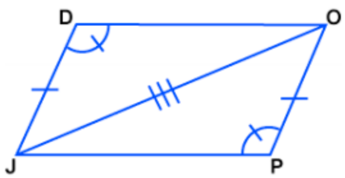
3. **REINFORCE** Suppose ΔPQR ≅ ΔPSR.

   a. Write out the congruence statements indicating which corresponding sides of the two triangles are congruent.

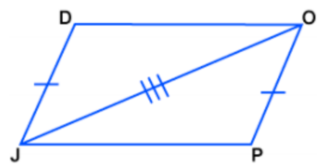
   b. Sketch the two congruent triangles.

4. **REINFORCE**

a. Given two triangles with corresponding angles and sides as marked congruent on this diagram, why is it not possible to conclude that the two triangles are congruent?




b. Add one set of tick marks to the diagram below to obtain a pattern in which the two triangles are congruent. Write out the congruence statement and indicate the congruence postulate you used.





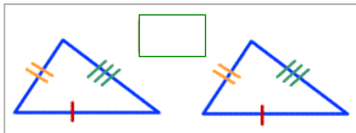
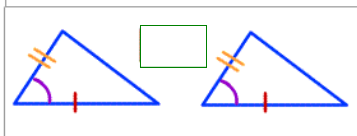
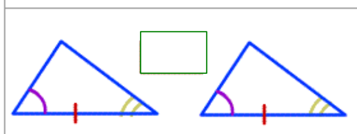
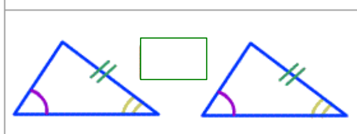
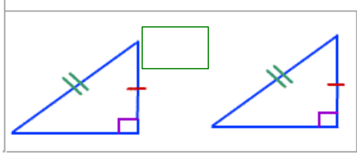



5. If two angles of one triangle are congruent to two angles of another triangle, what must be true about the third angles of the triangles? How do you know?

6. **REINFORCE** Suppose Δ**DOG** ≅ Δ**CAT**. If m∠**D** = 30° and m∠**A** = 50°, find m∠**T.** Explain your solution.

Complete the summary table below by filling in each blank with the correct abbreviation of the congruent triangle statements. Use the answer choices provided.

| SAA | ASA | SAS | SSS | HL |
|---|---|---|---|---|

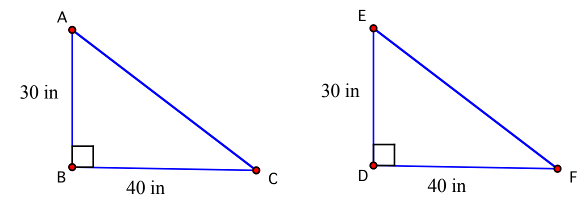
| | |
|---|---|
|  | If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. |
|  | If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. |
|  | If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. |
|  | If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of another triangle, then the triangles are congruent. |
|  | If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the triangles are congruent. |



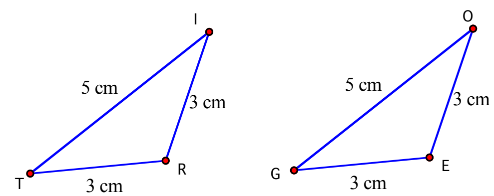


7. **REINFORCE** Consider each of the diagrams below, and decide whether you are given enough information to determine that the triangles are congruent. If so, write the congruence statement and the congruent triangle postulate you would use. If not, explain why not and draw a counterexample.

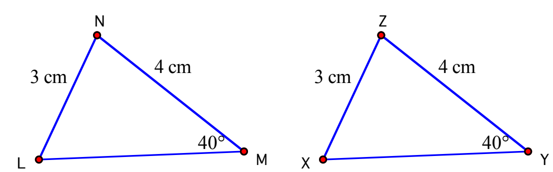
a. ____________________________________________________

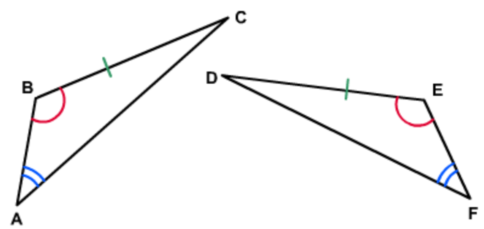



b. ____________________________________________________




c. ____________________________________________________






1. Are you given enough information to prove that these two triangles are congruent? If so, what reason would you give?

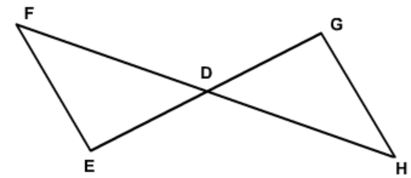
2. Complete the congruence statements that describe the two triangles above as congruent. Be sure to name the two triangles in order of their correspondence.

a. $\Delta$**ABC** ≅ __________ b. $\Delta$**CAB** ≅ __________

c. $\Delta$**CBA** ≅ __________ d. $\Delta$**BCA** ≅ __________





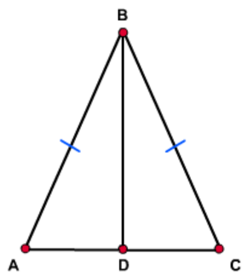



3. Consider the two triangles above. Suppose **D** is the midpoint of both $\overline{GE}$ and $\overline{FH}$. Are you given enough information to prove the triangles are congruent? If so, explain your reasoning.

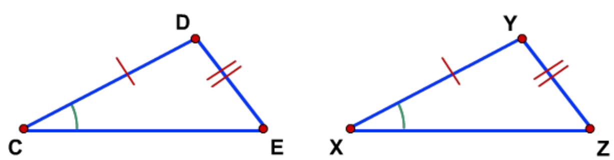
4. How would you complete the following congruence statement?

$\Delta$**FDE** ≅ __________

In this diagram, **D** is the midpoint of $\overline{AC}$ and $\overline{AB} \cong \overline{CB}$. Can you prove any triangles congruent? Explain your reasoning.




5. Consider the triangles in this diagram. Suppose you know that **CD** = **XY**, **DE** = **YZ**, and $\angle$**C** $\cong$ $\angle$**X**. Is this enough information to prove the triangles congruent? Explain your reasoning.

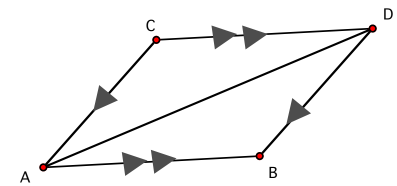




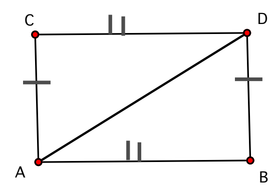



6. **REINFORCE** For each triangle pair below, decide if you can determine a triangle congruence from the given information. If so, write the triangle congruence statement and what postulate you can use. If not, explain why no congruence can be determined.

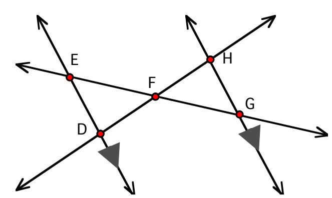
a. ______________________________________________________




b. ______________________________________________________

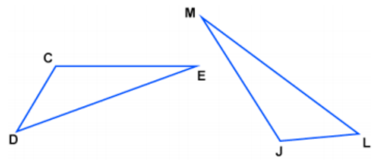



c. ______________________________________________________

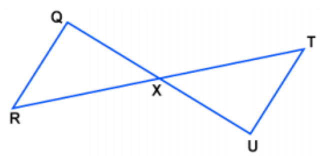


7. **REINFORCE** Use the given information, and other theorems and postulates you have learned, to decide if each pair of triangles can be proved congruent. If so, write the congruence statement and the triangle congruence postulate. You may want to mark the diagrams to show which parts are congruent.

a. Given: $\overline{CD} \cong \overline{JL}$, $\overline{CE} \cong \overline{JM}$, $\overline{DE} \cong \overline{LM}$




b. Given: $\overline{QU}$ and $\overline{RT}$ bisect each other.

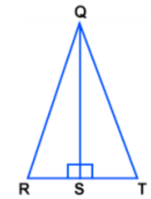




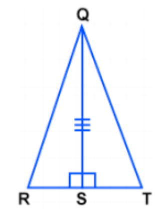



8. **REINFORCE** Prove that the following pairs of right triangles are congruent.

a. List the corresponding parts of each right triangle that must be congruent in order for **ΔQRS ≅ ΔQTS** by HL.




b. If the two right triangles are marked as indicated, what additional information is necessary in order to prove that **ΔQRS ≅ ΔQTS** by ASA?



Student:______________________________Class:______________Date___________

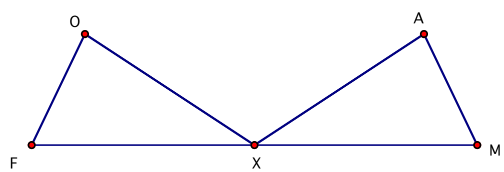
## Congruent triangle postulates

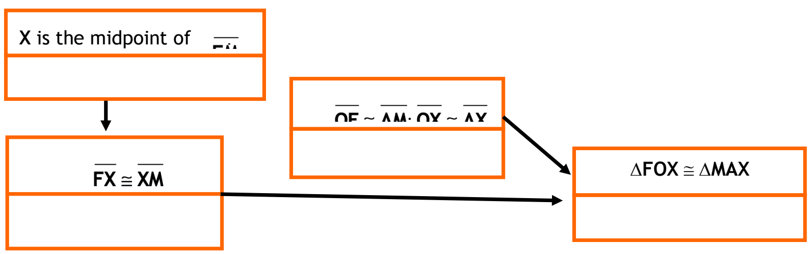


1. What are the four shortcuts you can use to prove two triangles congruent?

2. **REINFORCE** In the following proof, the statements have been filled in for you. Write the reason in the blank below each statement.

Given: X is the midpoint of $\overline{FM}$;
$\overline{OF} \cong \overline{AM}$; $\overline{OX} \cong \overline{AX}$.
Prove: $\Delta FOX \cong \Delta MAX$






Student:______________________________Class:______________Date___________

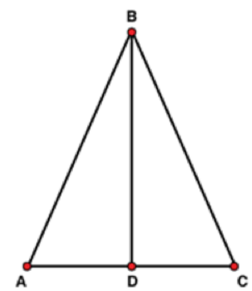
## Congruent triangle postulates

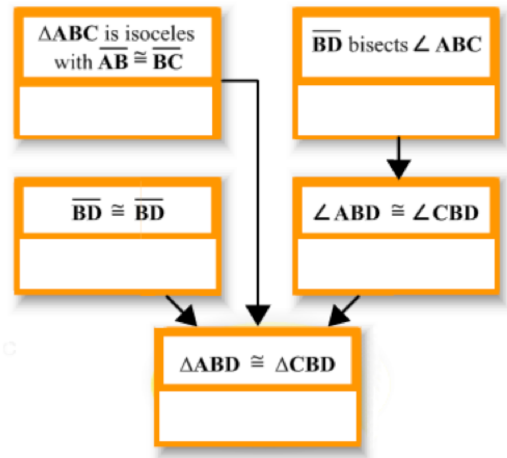


3. Complete the following proof by filling in the blanks in the flow chart.

**Given:** $\Delta ABC$ with $\overline{AB} \cong \overline{BC}$, and $\overline{BD}$ is the angle bisector of $\angle B$.

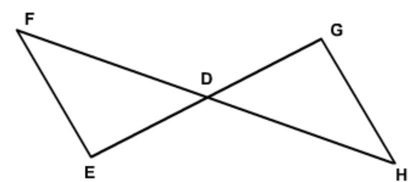
**Prove:** $\Delta ABD \cong \Delta CBD$




4. Write a proof of $\Delta FDE \cong \Delta HDG$, including all the statements and reasons.

Given: D is the midpoint of both $\overline{GE}$ and $\overline{FH}$.
Prove: $\Delta FDE \cong \Delta HDG$

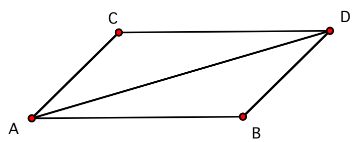


## Congruent triangle postulates



5. **REINFORCE** Write a proof of $\triangle ABD \cong \triangle DCA$. You may write a paragraph proof, a flow-chart proof, or a two-column proof.

Given: $\overline{AC} \parallel \overline{BD}$ ; $\overline{CD} \parallel \overline{AB}$
Prove: $\triangle ABD \cong \triangle DCA$





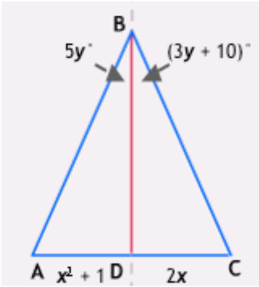




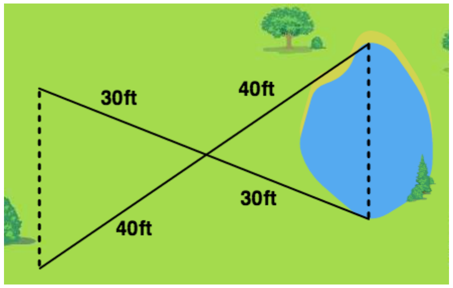
## Using congruent triangles



1. **REVIEW** Suppose $\triangle ABC$ is isosceles with **AB** = **BC** and altitude $\overline{BD}$. Find the length of **AC** and m$\angle$**ABC**.




2. In order to find the distance across a pond, a surveyor helps a park ranger measure the following distances to create two triangles. How can the park ranger be sure that the two triangles are congruent?



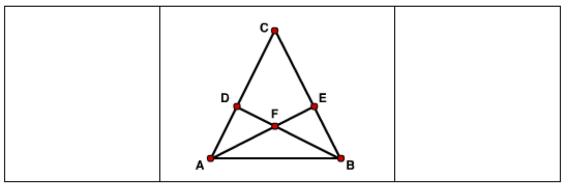
## Using congruent triangles

Topic 10 Student Activity Sheet 1; *Overview*



3. What does CPCTC stand for?

4. In the box to the left draw Δ**CEA**, and in the box to the right draw Δ**CDB**.





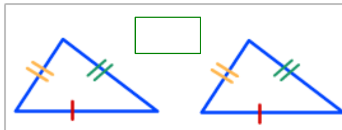
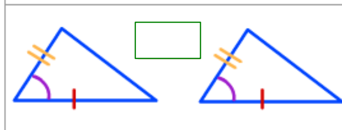
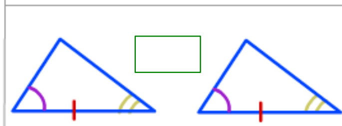
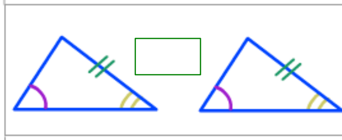
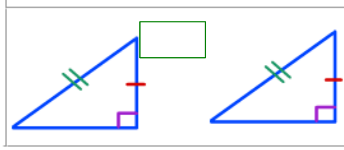



Using congruent triangles

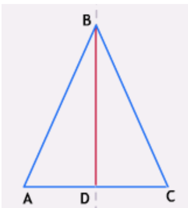
Topic 10 Student Activity Sheet 1; *Overview*



Fill in the blanks in the table with the abbreviations of the congruent triangle postulates.

| | |
|---|---|
|  | If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. |
|  | If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. |
|  | If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. |
|  | If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of another triangle, then the triangles are congruent. |
|  | If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the triangles are congruent. |

**REINFORCE** Suppose you want to prove $\overline{AD} \cong \overline{DC}$. These two segments are parts of which two triangles?



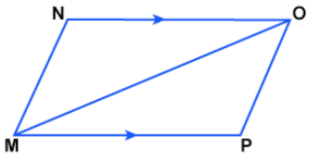




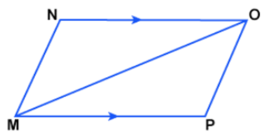


1. **REINFORCE** Given: $\overline{NO} \cong \overline{MP}$, $\overline{NO} \parallel \overline{MP}$
   Prove: $\angle N \cong \angle P$

   a. To complete this proof, first mark all the congruent parts on the diagram. Based on your markings, which triangles are congruent, and why? How can you use these triangles to prove $\angle N \cong \angle P$?




   b. Complete the proof by filling in the blanks.




   Given: $\overline{NO} \cong \overline{MP}$, $\overline{NO} \parallel \overline{MP}$
   Prove: $\angle N \cong \angle P$

| Statements | Reasons |
|---|---|
| 1. $\overline{NO} \parallel \overline{MP}$ | 1. Given |
| 2. $\angle NOM \cong \angle PMO$ | 2. |
| 3. $\overline{NO} \cong \overline{MP}$ | 3. |
| 4. $\overline{MO} \cong \overline{MO}$ | 4. |
| 5. $\Delta NOM \cong \Delta PMO$ | 5. |
| 6. $\angle N \cong \angle P$ | 6. |

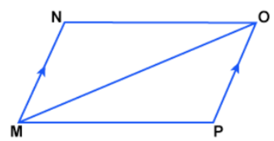
2. **REINFORCE** Complete the proof by choosing statements from the list below and filling in the steps of the proof in the correct order.

**Statements:**

| | | | | | | |
|---|---|---|---|---|---|---|
| $\overline{MO} \cong \overline{MO}$ | $\angle NOM \cong \angle PMO$ | $\overline{NO} \parallel \overline{MP}$ | $\angle NMO \cong \angle POM$ | $\angle N \cong \angle P$ | $\overline{NM} \parallel \overline{OP}$ | $\Delta NOM \cong \Delta PMO$ |

Given: $\overline{NM} \parallel \overline{OP}$, $\angle N \cong \angle P$

Prove: $\overline{NO} \parallel \overline{MP}$



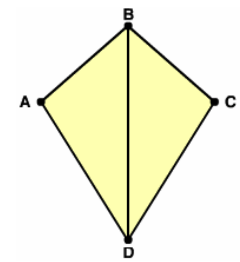

| Statement | Reasons |
|---|---|
| 1. $\overline{NM} \parallel \overline{OP}$ | 1. Given |
| 2. | 2. If parallel lines are cut by a transversal, then alternate interior angles are congruent. |
| 3. | 3. Given |
| 4. | 4. Reflexive property of congruence |
| 5. | 5. AAS |
| 6. | 6. CPCTC |
| 7. | 7. If the alternate interior angles are congruent, then the lines are parallel. |






3. Mark the congruent parts of the triangles and complete the proof.

| | | | | |
|---|---|---|---|---|
| SAS | Reflexive | $\Delta ABD \cong \Delta CBD$ | Def $\angle$ bis. | ASA |
| $\angle A \cong \angle C$ | $\Delta ABD \cong \Delta DBC$ | CPCTC | Given | Symmetric |




| Statements | Reasons |
|---|---|
| 1. $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$ | 1. |
| 2. $\angle ABD \cong \angle CBD$; $\angle ADB \cong \angle BDC$ | 2. |
| 3. $\overline{BD} \cong \overline{BD}$ | 3. property |
| 4. | 4. |
| 5. | 5. |

Student:________________________________________Class:_______________Date___________

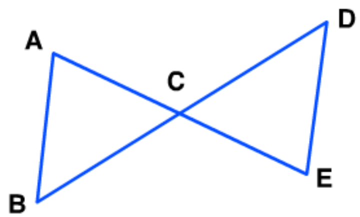
## Using congruent triangles



4. Mark the congruent parts of the triangles and complete the proof.

| | | | | |
|---|---|---|---|---|
| Given | ASA Postulate | $\triangle BCA \cong \triangle DCE$ | CPCTC | $\angle ACB \cong \angle ECD$ |
| $\overline{AE}$ bisects $\overline{BD}$ | $\overline{BC} \cong \overline{CD}$ | $\overline{AC} \cong \overline{CE}$ | $\overline{DB}$ bisects $\overline{AE}$ | |

**Given:** $\angle B \cong \angle D$; $\overline{AE}$ bisects $\overline{BD}$
**Prove:** $\overline{DB}$ bisects $\overline{AE}$




| Statements | Reasons |
|---|---|
| $\angle B \cong \angle D$ | |
| | Given |
| | Def. of segment bisector |
| | Vertical angles are congruent. |
| | |
| | |
| | Def. of segment bisector |

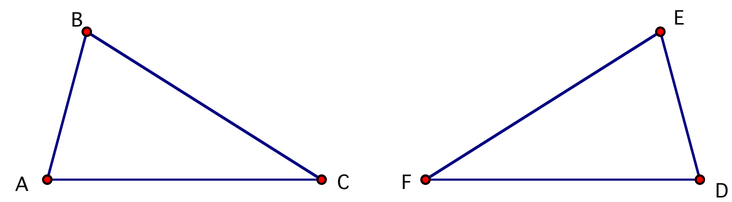


Student:________________________________________Class:_______________Date___________

## Using congruent triangles



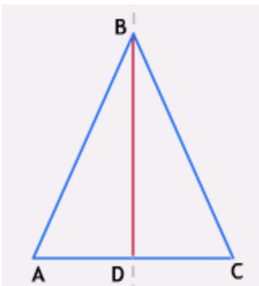
5. **REINFORCE** Given $\triangle ABC \cong \triangle DEF$, name all of the corresponding parts you could prove congruent using CPCTC.



## Using congruent triangles



1. List the isosceles triangles conjectures you made in the topic **Properties of a Triangle.**



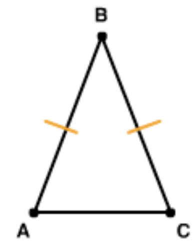

2. **REINFORCE** If the vertex angle of an isosceles triangle has a measure of 50°, what are the measures of the two base angles? Explain your solution.





## Using congruent triangles



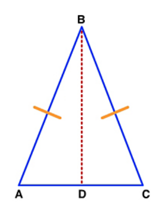


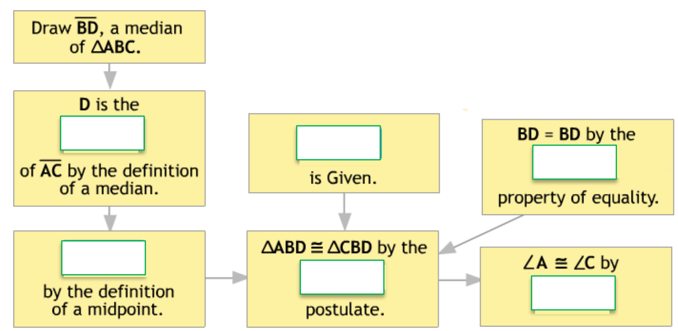

3. Complete this proof of the Isosceles Triangle Theorem.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **AD = DC** | **CPCTC** | **reflexive** | **AB = BC** | **symmetric** | **SSA** | **midpoint** | **SSS** |

**Given**: AB = BC

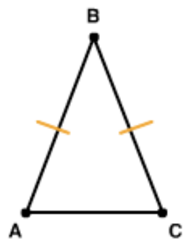
**Prove** ∠A ≅ ∠C



## Using congruent triangles






a. Decide if each of the following triangle congruence statements is true or false. Explain your reasoning.

__________ΔABC ≅ ΔABC

__________ΔABC ≅ ΔCBA

__________ΔABC ≅ ΔBAC

b. Which one of the true triangle congruency statements above can be used to prove the Isosceles Triangle Theorem? Explain your reasoning.





## Using congruent triangles

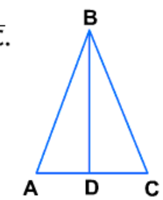


4. Parts a and b below guide the proofs for the remaining isosceles triangle conjectures.

   a. Complete the proof of the third conjecture: $\overline{BD}$ bisects ∠ABC.

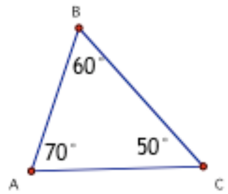
| | | | | | | |
|---|---|---|---|---|---|---|
| CPCTC | ΔDAB ≅ ΔDBC | $\overline{BD}$ bisects ∠ABC | Definition of midpoint | ΔABD ≅ ΔCBD | Given | SSS |

**Given:** AB = BC and $\overline{BD}$ is the median of $\overline{AC}$.
**Prove:** $\overline{BD}$ bisects ∠ABC.




| Statements | Reasons |
|---|---|
| 1. AB = BC | 1. |
| 2. $\overline{BD}$ is the median of $\overline{AC}$. | 2. |
| 3. | 3. Median of isos. △ forms two ≅ △s. |
| 4. ∠ABD ≅ ∠CBD | 4. |
| 5. | 5. Def. of ∠ bisector |

9. **REINFORCE** Given the triangle below with the angle measures shown, rank the side lengths in order from smallest to greatest.




_____ < _____ < _____

4/8/20:

1. REVIEW Look at the patterns below. Can you find the next two items in each list and state the rule for finding them?

   a. 2, 4, 6, 8, ...

      10, 12
      Rule: Add 2.

   b. 2, 3, 5, 9, 17, ...

      33, 65
      Rule: Multiply by 2 then subtract 1.

   c. 1, -2, 3, -4, 5, ...

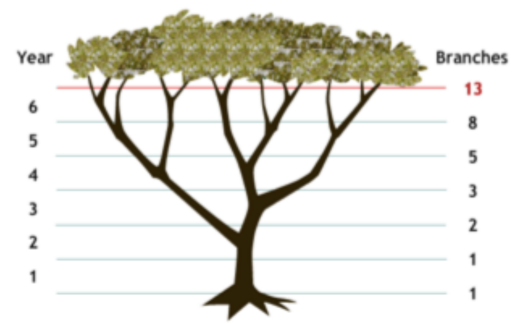
      -6, 7
      Rule: Add 1 but alternate the signs.

   d. 1, 4, 9, 16, ...

      25, 36
      Rule: Count up starting with 1, squaring each integer.








6. In the diagram, the numbers of branches that appear as the tree grows model the ***Fibonacci numbers***. This sequence of numbers is named after the Italian mathematician Leonardo Fibonacci (1170-1250 AD). Can you find the pattern in the number of branches as the tree grows? **[OV, page 1]**

   The first two numbers are 1. Subsequently, you add the two previous numbers to find the next number in the pattern.

7. Fill in the next few numbers in the sequence of Fibonacci numbers. **[OV, page 1]**

   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

## Using inductive reasoning and conjectures



9. In the picture below, find as many geometric objects as you can. Make a list of all the geometric objects that you find. **[OV, page 3]**

Sample answers:

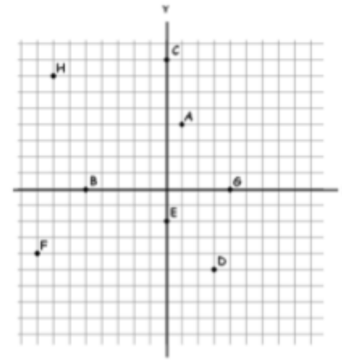
- Points
- Line segments
- Angles
- Polygons
- Rectangular prisms





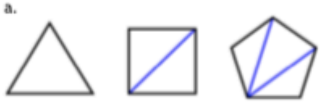

11. **REVIEW** Graph and label the following points on the coordinate plane.

**A** (1,4)
**B** (-5,0)
**C** (0,8)
**D** (3,-5)
**E** (0,-2)
**F** (-8,-4)
**G** (4,0)
**H** (-7,7)



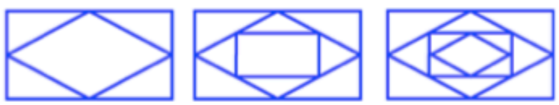
12. **REINFORCE** Draw the next image for each of these patterns. Write a description of the rule represented by the image and explain how to use the rule to find the next figure.

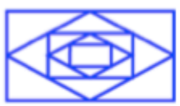
a.



The pattern shows that each image in the pattern has one more side than the previous image and one additional diagonal than the previous image from the same vertex. To find the next image in the pattern, add a side of equal length to the polygon and add a diagonal from the common vertex to the new vertex in the polygon.

b.





Each image in this pattern has a quadrilateral formed inside of the inner most quadrilateral of the previous image. The new quadrilateral is formed by connecting a point in the middle of each side of the inner most quadrilateral. To find the next image in the pattern, find a point in the middle of each of the four sides of the inner most quadrilateral and connect those points to create another quadrilateral.





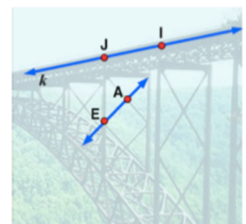

4/9/20:

**REINFORCE** Find a geometric representation for the following sequence of numbers.

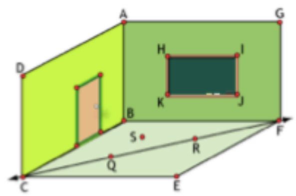
3, 4, 5, 6, 7, …

Student answers will vary. For example, students could draw a sequence of polygons with the number of sides representing the terms in the sequence.

What are two names for the line containing points **A** and **E**? **[EX1, page 1]**




The line can be named $\overleftrightarrow{AE}$ or $\overleftrightarrow{EA}$.




14. **Reinforce**

a. Name two points in the room diagram that are collinear with points **C** and **F**. **[EX1, page 2]**

Points **Q** and **R** are collinear with **C** and **F**.

b. Point **J** is noncollinear with points **H** and **K**. Name another point that is noncollinear with points **H** and **K**. **[EX1, page 2]**

Any other point in the diagram is noncollinear with **H** and **K**.

c. Points **C**, **Q**, and **S** are coplanar points. Name another point on the floor that is coplanar with **C** and **Q**. **[EX1, page 2]**

Points **B**, **R**, **F**, and **E** are all coplanar with **C** and **Q**.

d. Points **A**, **B**, and **F** are noncoplanar with point **C**. Name another point in the room that is noncoplanar with **A**, **B**, and **F**. **[EX1, page 2]**

Points **D**, **Q**, **R**, **S**, and **E** are all noncoplanar with **A**, **B**, and **F**.





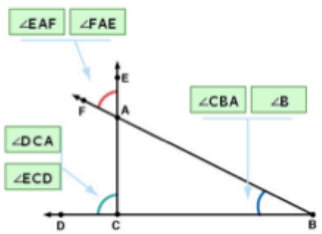

15. Using the notations provided, complete the table by writing in the correct notation for each name and figure. **[EX1, page 2]**

| | | | |
|---|---|---|---|
| AB | BA | $\overline{AB}$ | $\overline{BA}$ |
| $\overrightarrow{AB}$ | $\overrightarrow{BA}$ | $\overleftrightarrow{AB}$ | $\overleftrightarrow{BA}$ |

| Figure | Name | Notation |
|---|---|---|
|  | Line AB | $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ |
|  | Ray AB | $\overrightarrow{AB}$ |
|  | Ray BA | $\overrightarrow{BA}$ |
|  | Segment AB | $\overline{BA}$ or $\overline{AB}$ |
|  | The distance between A and B | AB or BA |

16. Using the angle names provided, label the angles in the diagram below. **[EX1, page 3]**

| ∠B | ∠EAF | ∠A | ∠ECD | ∠FEA | ∠FAE | ∠CBA | ∠DCA |
|---|---|---|---|---|---|---|---|




17. Write a definition of supplementary angles. Give an example of two supplementary angles. **[EX1, page 4]**

    Two angles are supplementary if the sum of their measures is 180°. A 130° angle and a 50° angle are supplementary angles.

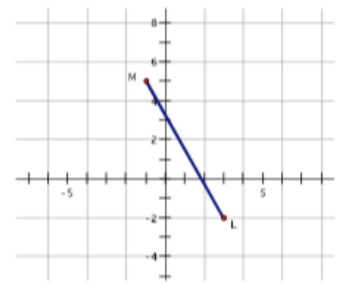
18. Write a definition of complementary angles. Give an example of two complementary angles. **[EX1, page 4]**

    Two angles are complementary if the sum of their measures is 90°. A 40° angle and a 50° angle are complementary angles.





20. **REINFORCE** Draw and label $\overline{LM}$ where **L** has coordinates (3,-2) and **M** has coordinates (-1,5).




21. **REINFORCE** Suppose ∠**A** and ∠**B** are complementary angles, m∠**A** = (3$x$ + 5)°, and m∠**B** = (2$x$ - 15)°. Solve for $x$ and then find m∠**A** and m∠**B**.

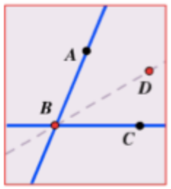
    $(3x + 5) + (2x - 15) = 90$
    $5x - 10 = 90$
    $5x = 100$
    $x = 20$
    m∠**A** = (3(20) + 5)° = 65°
    m∠**B** = (2(20) - 15)° = 25°

22. **REINFORCE** The measure of the supplement of an angle is 12 more than twice the measure of the angle. Find the measures of the angle and its supplement.

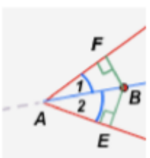
    Let $x$ = the angle measure. Then 180 – $x$ is the measure of the supplement.
    $180 - x = 12 + 2x$
    $168 = 3x$
    $x = 56$
    $180 - x = 124$
    The measures of the angle and its supplement are 56° and 124°.

23. Write a definition for ***angle bisector***, and then sketch an example. **[EX1, page 6]**

An angle bisector is a line, ray, or segment in the plane of an angle that divides the angle into two congruent angles.




28. **REINFORCE** In the diagram, $\overrightarrow{AB}$ bisects $\angle FAE$. $BF = 5x$ and $BE = x^2 + 6$. Solve for $x$.




Because B is on the angle bisector of $\angle FAE$, $BF = BE$.

$x^2 + 6 = 5x$

$x^2 - 5x + 6 = 0$

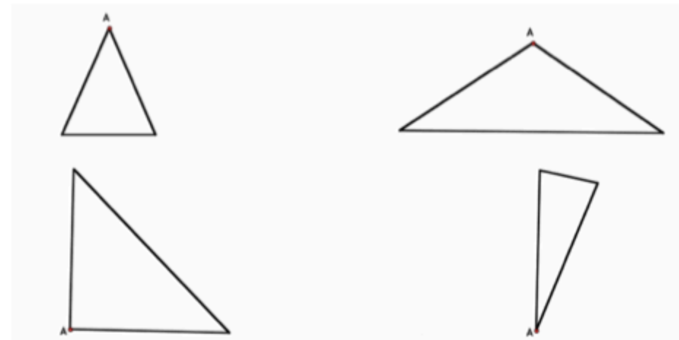
$(x - 2)(x - 3) = 0$

$x = 2$ or $x = 3$





31. **REINFORCE** Below are several isosceles triangles. Construct the angle bisector of $\angle A$ on each triangle. Then write a conjecture about the angle bisector of the angle formed by the two congruent sides of an isosceles triangle.



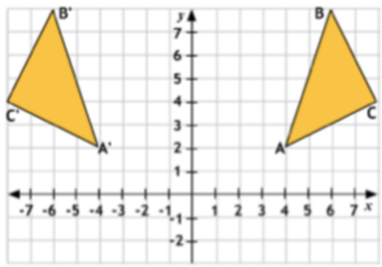

Student conjectures may vary. *Note to teacher: students may not use precise language at this point in the course. Accept their conjectures, but help them begin to use more precise mathematical language. As this is an early topic, you can establish an expectation for precise language in this course. Some students may want to use terms that have not yet been covered. Share these terms and let students know that they will study more about these terms in the course.*

Some sample conjectures:

The angle bisector of the angle formed by the two congruent sides of an isosceles triangle cuts the triangle into two congruent triangles.
The angle bisector of the angle formed by the two congruent sides of an isosceles triangle intersects the third side at the midpoint of that side.
The angle bisector of the angle formed by the two congruent sides of an isosceles triangle is perpendicular to the third side.

4/10/20:

2. Reflect $\Delta$**ABC** across the ***y***-axis. **[OV, page 3]**

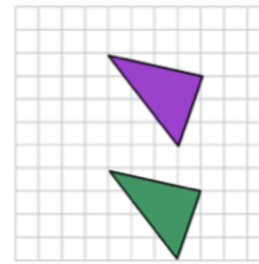




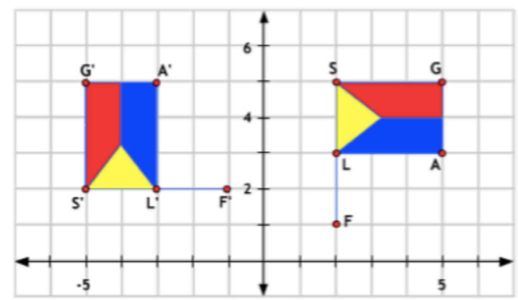




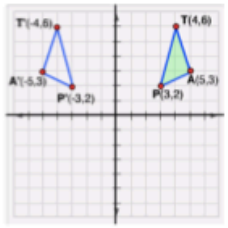
3. Translate the triangle down 5 units. **[OV, page 3]**



4. Rotate **FLAGS** 90° counterclockwise about the origin. **[OV, page 3]**



4/13/20:




1. Using Patty Paper, reflect ΔPAT across the *y*-axis. Label the image ΔP'A'T'. What are the coordinates of the vertices of ΔP'A'T'? **[EX1, page 2]**

   *Answer shown on graph above.*

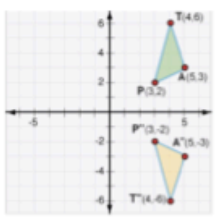
2. Write a conjecture about what happens to the coordinates of a point when you reflect it across the *y*-axis. **[EX1, page 2]**

   When you reflect a point across the *y*-axis, the *x*-coordinates change signs.

3. Write a conjecture about what happens to the coordinates of a point when you reflect it across the *x*-axis. **[EX1, page 3]**

   When you reflect a point across the *x*-axis, the *y*-coordinates change signs.

4. Using Patty Paper, reflect ΔPAT across the *x*-axis. Label the image ΔP''A''T''. What are the coordinates of the vertices of ΔP''A''T''? **[EX1, page 3]**









| | | | |
|---|---|---|---|
| *x* | *y* | -*x* | -*y* |

5. Use the answer choices shown above to complete the following statements. **[EX1, page 4]**

   a. A reflection across the *x*-axis maps the point (*x*,*y*) to the point (*x*,-*y*).

   b. A reflection across the *y*-axis maps the point (*x*,*y*) to the point (-*x*,*y*).

6. Using ordered pair rule notation, rewrite the rules you completed in question 5. **[EX1, page 5]**

   a. (*x*,*y*) → (*x*,-*y*)

   b. (*x*,*y*) → (-*x*,*y*)

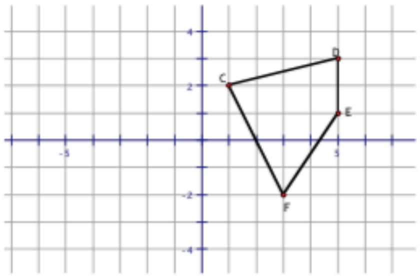
8. Compare the coordinates of **C** and **C'**, **O** and **O'**, and **T** and **T'**. Notice what is true about the coordinates of each pre-image (*x*,*y*) and its image (*x'*,*y'*). Then use the given answer choices to complete the following statements. **[EX1, page 7]**

| | | | |
|---|---|---|---|
| *x* | *y* | stay the same | have their signs changed |
| -*x* | -*y* | are interchanged | |

   a. When the pre-image (*x*,*y*) is reflected across the line *y* = *x*, the *x*- and *y*-coordinates of the pre-image and image are interchanged.

   b. The ordered pair rule for a reflection across the line *y* = *x* is

   (*x*,*y*) → (*y*,*x*).




9. **REINFORCE** Quadrilateral **CDEF** has the following vertices: **C**(1,2), **D**(5,3), **E**(5,1), and **F**(3,-2).

   a. Plot quadrilateral **CDEF** on the grid.

   *Answer shown on graph above.*

   b. Reflect quadrilateral **CDEF** across the ***x***-axis. What are the coordinates of the image?

   **C'**(1,-2), **D'**(5,-3), **E'**(5,-1), **F'**(3,2)

   c. Reflect quadrilateral **CDEF** across the ***y***-axis. What are the coordinates of the image?

   **C'**(-1,2), **D'**(-5,3), **E'**(-5,1), **F'**(-3,-2)

   d. Reflect quadrilateral **CDEF** across the line ***y*** = ***x***. What are the coordinates of the image?

   **C'**(2,1), **D'**(3,5), **E'**(1,5), **F'**(-2,3)





11. Using the answer choices provided, name the transformation that goes with each ordered pair rule. Assume $a \neq b$. **[EX1, page 10]**

| | | |
|---|---|---|
| **reflection across *y* = *x*** | **reflection across the *x*-axis** | **reflection across the *y*-axis** |
| **rotation of 180° about (0,0)** | **doesn't match a given transformation** | |

   a. $(a,b) \rightarrow (b,a)$ reflection across *y* = *x*

   b. $(a,b) \rightarrow (-a,b)$ reflection across the *y*-axis

   c. $(a,b) \rightarrow (a,-b)$ reflection across the *x*-axis

   d. $(a,b) \rightarrow (-a,-b)$ rotation of 180° about (0,0)

   e. $(a,b) \rightarrow (-b,-a)$ doesn't match a given transformation

12. **REINFORCE** Find the image of the point (5,8) for each transformation described.

a. Reflection across the ***x***-axis.

(5,-8)

b. Reflection across the ***y***-axis.

(-5,8)

c. Reflection across the line ***y*** = ***x***.

(8,5)

d. Reflection across the ***x***-axis, followed by a reflection across the ***y***-axis.

(-5,-8)

e. Reflection across the ***y***-axis, followed by a reflection across the ***x***-axis.

(-5,-8)

f. Rotation about the origin by 180°.

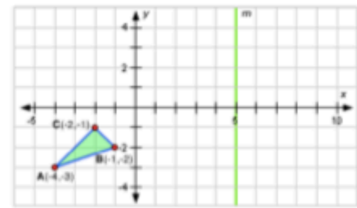
(-5,-8)

4/14/20:





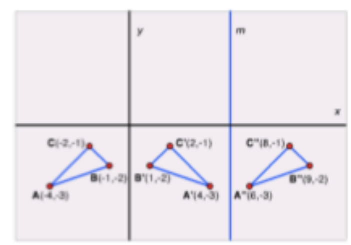
1. Use this grid to complete the Patty Paper exercise below. **[EX2, page 2]**



- Overlay your Patty Paper to copy the ***x***- and ***y***-axes, line ***m***, and **ΔABC**.
- Reflect **ΔABC** across the ***y***-axis. Mark the locations of **A'**, **B'**, and **C'**.
- Overlay your Patty Paper on the grid to find the coordinates of **A'**, **B'**, and **C'**.
- Reflect **ΔA'B'C'** across the line ***x*** = ***m***. Mark the locations of **A"**, **B"**, and **C"**.
- Overlay your Patty Paper on the grid to find the coordinates of **A"**, **B"**, and **C"**.

Sketch the results from your Patty Paper on the grid, or attach your Patty Paper to this Student Activity Sheet.

*Completed Patty Paper activity should look like this:*

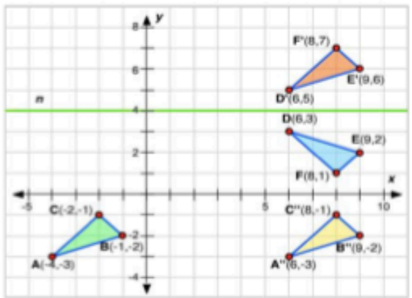


5. Complete the steps below to investigate what happens when $\Delta A''B''C''$ translates vertically. **[EX2, pages 5 and 6]**

**Step 1:** Draw $\Delta A''B''C''$ and the line $y = 4$ on the coordinate grid. Label the line $n$.

**Step 2:** Draw the reflection image of $\Delta A''B''C''$ across the $x$-axis. Label the reflection image of $\Delta A''B''C''$ and record the coordinates of the vertices D, E, and F.

**Step 3:** Now reflect $\Delta DEF$ across line $n$ to get the translation image of $\Delta A''B''C''$. Label the translation image $\Delta D'E'F'$ and record the coordinates of the vertices.




6. What is the ordered pair rule for reflecting A" twice to D'? **[EX2, page 7]**

Point A" moved to D and D' in the following way: (6,-3) → (6,3) → (6,5).

7. What are the single ordered pair rules for translating points A", B", and C"? **[EX2, page 7]**

The single ordered pair rule for translating point A" to D' is (6,-3) → (6,5). The rule for B" to E' is (9,-2) → (9,6) and the rule for C" to F' is (8,-1) → (8,7).

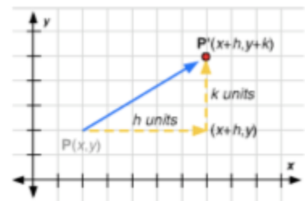
8. To compare the beginning position of ΔABC with the ending position of ΔD'E'F', write single ordered pair rules for corresponding vertices. Then describe the translation in words. **[EX2, page 7]**

A(-4,-3) → D'(6,5); B(-1,-2) → E'(9,6); C(-2,-1) → F'(8,7)
Each vertex of ΔABC is translated 10 units horizontally and 8 units vertically.






10. Use vector notation to describe the vector on the graph. **[EX2, page 9]**




$P \xrightarrow{\langle h,k \rangle} P'$

11. Using the answer choices provided, complete the following statements. **[EX2, page 11]**

| | | | |
|---|---|---|---|
| $k < 0$ | $h > 0$ | $x + h$ | $x - h$ |
| $k > 0$ | $h < 0$ | $y + k$ | $y - k$ |

a. If P($x$,$y$) is translated $|h|$ units to the right, the signed value of $h$ is $h > 0$.
b. If P($x$,$y$) is translated $|h|$ units to the left, the signed value of $h$ is $h > 0$.
c. If P($x$,$y$) is translated $|k|$ units up, the signed value of $k$ is $k > 0$.
d. If P($x$,$y$) is translated $|k|$ units down, the signed value of $k$ is $k < 0$.
e. A single ordered pair rule for translating P($x$,$y$) $h$ units horizontally and $k$ units vertically is $P(x,y) \xrightarrow{\langle h,k \rangle} P'(x+h, y+k)$.

12. **REINFORCE** A point has coordinates ($x$,$y$). Write an ordered pair rule for a translation that moves the point 5 units to the right and 3 units down.

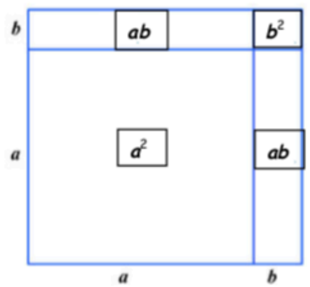
$(x,y) \rightarrow (x+5, y-3)$ or $(x,y) \xrightarrow{\langle 5,-3 \rangle} (x+5, y-3)$

4/15/20:

| | | |
|---|---|---|
| $b^2$ | $ab$ | $a^2$ |

1. Use the answer choices above to fill in the blanks in the diagram and show why the mathematical statement is true. **[EX1, page 1]**

$$(a + b)(a + b) = a^2 + 2ab + b^2$$




2. Solve the following equation. As you complete each step in the solution, take time to think about why that step is true. Show all of your work. **[EX1, page 3]**

$4x + 2 = 10$

$4x + 2 - 2 = 10 - 2$

$4x = 8$

$\frac{4x}{4} = \frac{8}{4}$





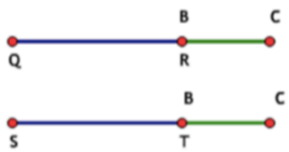

**Properties of Equality**
For any real numbers ***a***, ***b***, and ***c***:

| Addition Property | Substitution Property |
|---|---|
| If $a = b$, then $a + c = b + c$. | If $a = b$, then $b$ can substitute for $a$ in any equation. |
| **Subtraction Property** | **Reflexive Property** |
| If $a = b$, then $a - c = b - c$. | $a = a$ |
| **Multiplication Property** | **Symmetric Property** |
| If $a = b$, then $ac = bc$. | If $a = b$, then $b = a$. |
| **Division Property** | **Transitive Property** |
| If $a = b$, then $\frac{a}{c} = \frac{b}{c}$; $c \neq 0$. | If $a = b$, and $b = c$, then $a = c$. |

3. When the properties are listed alongside an equation so that each step is justified, the result is an algebraic proof. Use the properties of equality to complete the justification of the solution to the equation you solved in question 2. **[EX1, page 7]**

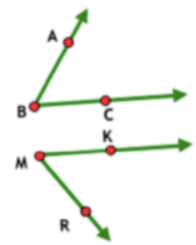
| Statements | Reasons |
|---|---|
| $4x + 2 = 10$ | Given equation |
| $4x + 2 - 2 = 10 - 2$ | Subtraction Property of Equality |
| $4x = 8$ | Simplify |
| $\frac{4x}{4} = \frac{8}{4}$ | Division Property of Equality |
| $x = 2$ | Simplify |

1. Use a property of equality to justify each of the following statements. **[EX1, page 8]**




If **QR** = **ST**, then
**QR** + **BC** = **ST** + **BC**.

a. Addition Property




If $m\angle ABC = m\angle KMR$,
then $m\angle KMR = m\angle ABC$.

b. Symmetric Property

**AX** = **AX**

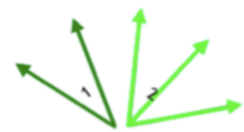
c. Reflexive Property


If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$,
then $m\angle 1 = m\angle 3$.

d. Transitive Property





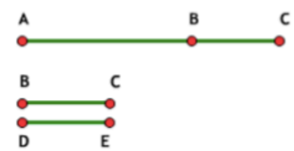



If $2 \cdot m\angle 1 = m\angle 2$,
then $m\angle 1 = \frac{m\angle 2}{2}$.

e. Division Property

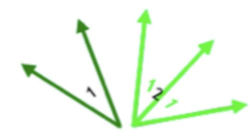

If **JK** + **AB** = **RQ** + **AB**,
then**JK** = **RQ**.

f. Subtraction Property




From the diagram, **AB** + **BC** = **AC**.
If **BC** = **DE**, then **AB** + **DE** = **AC**.

g. Substitution Property




If $m\angle 1 = \frac{1}{2} \cdot m\angle 2$,
then $2 \cdot m\angle 1 = m\angle 2$.

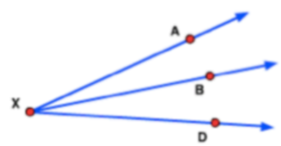
h. Multiplication Property

4/20/20:

2. Points **A**, **Z**, and **B** lie on $\overline{AB}$. If **AZ** = 2 centimeters and **ZB** = 3 centimeters, what is **AB**? Explain how you found your answer. **[EX2, page 2]**


**AB** = 5 centimeters. It may seem clear that **AZ** + **ZB** is equal to the length **AB**; however, this solution is based on the assumption that **A**, **Z**, and **B** are all on a line and **Z** is between **A** and **B**.

3. Now consider a similar question involving angles. Given the angles shown in the diagram, if $m\angle AXB = 15^\circ$ and $m\angle BXD = 20^\circ$, what is $m\angle AXD$? On what assumptions are you basing your answer? **[EX2, page 2]**




$m\angle AXD = 35^\circ$. This answer is based on the assumption that $\angle AXD$ and $\angle BXD$ share a side and a vertex and **B** is inside $\angle AXD$.





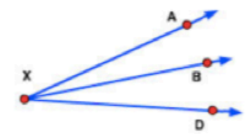
4. Use the given answer choices to complete the statements. **[EX2, page 3]**

| ZB | m∠BXD | m∠AXD | AB | m∠AXB | AZ |
|---|---|---|---|---|---|

**Segment Addition Postulate:**
If **Z** is between **A** and **B**, then


**AZ** + **ZB** = **AB**.

**Angle Addition Postulate:**
If **B** is in the interior of $\angle AXD$, then

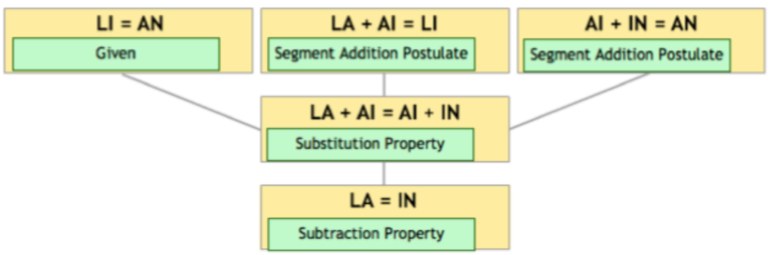



$m\angle AXB + m\angle BXD = m\angle AXD$.

8. Using the answer choices provided, fill in the correct reasons for each of the statements in this flow-chart proof. **[EX2, page 7]**

| | | | |
|---|---|---|---|
| Substitution Property | Division Property | Angle Addition Property | Multiplication Property |
| Subtraction Property | Segment Addition Postulate | Addition Property | Given |

Given: LI = AN
Prove: LA = IN






9. Using the answer choices provided, fill in the correct reasons for each of the statements in this two-column proof. **[EX2, page 8]**

| | | | | |
|---|---|---|---|---|
| AI = AN | LI = AN | LA = IN | AI = LI | LA + AI = IN + AI |

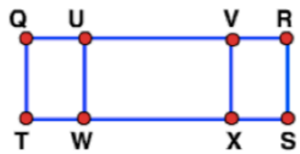
Given: LA = IN
Prove: LI = AN

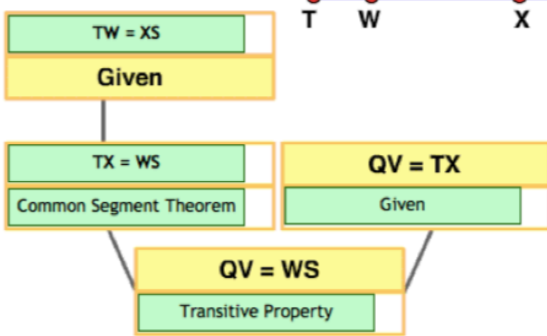

| STATEMENTS | REASONS |
|---|---|
| 1. LA = IN | 1. Given |
| 2. LA + AI = IN + AI | 2. Addition Property of Equality |
| 3. LA + AI = LI | 3. Segment Addition Postulate |
| 4. IN + AI = AN | 4. Segment Addition Postulate |
| 5. LI = AN | 5. Substitution Property |

12. Use the given answer choices to fill in both statements and reasons in the flow-chart proof. **[EX2, page 11]**

| | | |
|---|---|---|
| Transitive Property | Given | TX = WS |
| TW = XS | Common Segment Theorem | Segment Addition Postulate |

**Given:** TW = XS,
QV = TX
**Prove:** QV = WS





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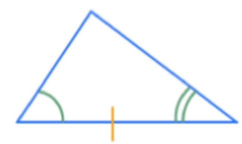
6. On the diagram, two sides are marked. Mark the angle that is considered the **included angle** in relation to these two sides. **[EX1, page 6]**

*(An included angle is an angle formed by two rays that coincide with the two sides.)*

7. On the diagram, two angles are marked. Mark the side that is considered the **included side** in relation to these two angles. **[EX1, page 6]**



*(The included side coincides with the ray shared by both angles.)*

8. Put an "X" in the table to indicate which combinations of three pairs of congruent parts guarantee two triangles congruent. **[EX1, pages 8 and 9]**

| | Proves congruence | Does not prove congruence |
|---|---|---|
| SSS | X | |
| AAA | | X |
| SAS | X | |
| SSA | | X |
| ASA | X | |
| SAA | X | |

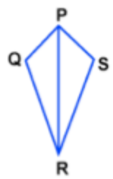
9. **Reinforce** Suppose $\Delta PQR \cong \Delta PSR$.

a. Write out the congruence statements indicating which corresponding sides of the two triangles are congruent.

$\overline{PQ} \cong \overline{PS}$
$\overline{QR} \cong \overline{SR}$
$\overline{PR} \cong \overline{PR}$

b. Sketch the two congruent triangles.

Student drawings may vary, but all sketches should show the triangles sharing a common side. Two examples are given below.

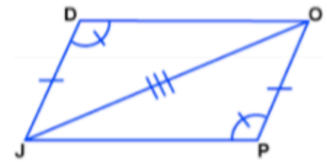






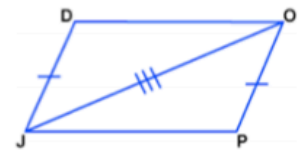
10. **Reinforce**

a. Given two triangles with corresponding angles and sides as marked congruent on this diagram, why is it not possible to conclude that the two triangles are congruent?

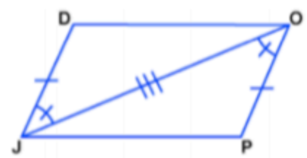



These two triangles have corresponding angles and sides marked congruent in the SSA pattern, which is a pattern that does not guarantee triangle congruence.

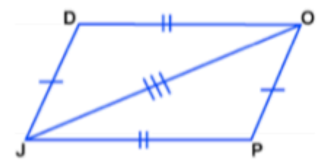
b. Add one set of tick marks to the diagram below to obtain a pattern in which the two triangles are congruent. Write out the congruence statement and indicate the congruence postulate you used.




There are two possible answers.




$\Delta DJO \cong \Delta POJ$ by SAS




$\Delta DJO \cong \Delta POJ$ by SSS

11. If two angles of one triangle are congruent to two angles of another triangle, what must be true about the third angles of the triangles? How do you know? **[EX1, page 9]**

    The third angles of the triangles must also be congruent. You can show this by using the Triangle Sum Theorem. If two angles of each triangle have measures $x°$ and $y°$, then the third angle in each triangle must have a measure of $180° - (x° + y°)$.

12. **REINFORCE** Suppose $\Delta DOG \cong \Delta CAT$. If $m\angle D = 30°$ and $m\angle A = 50°$, find $m\angle T$. Explain your solution.

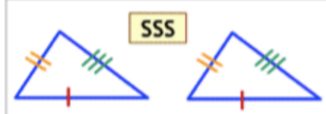
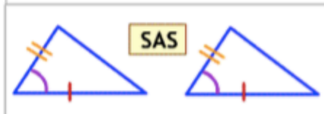
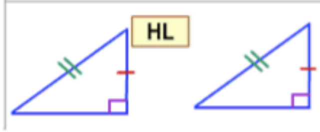
    Because the triangles are congruent, all of their corresponding angles are congruent. So, $m\angle C = 30°$. Using the Triangle Sum Theorem, $30° + 50° + m\angle T = 180°$. Therefore, $m\angle T = 100°$.





14. Complete the summary table below by filling in each blank with the correct abbreviation of the congruent triangle statements. Use the answer choices provided. **[EX1, page 12]**

| SAA | ASA | SAS | SSS | HL |
|---|---|---|---|---|

| | |
|---|---|
|  SSS | If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. |
|  SAS | If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. |
|  ASA | If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. |
|  SAA | If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of another triangle, then the triangles are congruent. |
|  HL | If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the triangles are congruent. |

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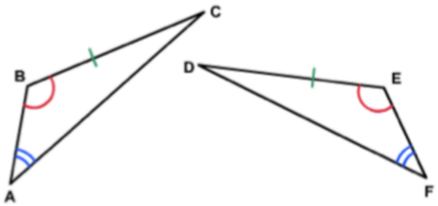



1. Are you given enough information to prove that these two triangles are congruent? If so, what reason would you give? **[EX2, page 2]**

   The diagram shows that two pairs of corresponding angles and the pair of non-included sides are congruent. You can use the SAA postulate to prove that these two triangles are congruent.

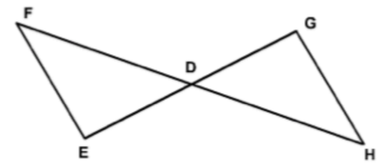
2. Complete the congruence statements that describe the two triangles above as congruent. Be sure to name the two triangles in order of their correspondence. **[EX2, page 2]**

   a. $\Delta ABC \cong \Delta FED$  b. $\Delta CAB \cong \Delta DFE$

   c. $\Delta CBA \cong \Delta DEF$  d. $\Delta BCA \cong \Delta EDF$







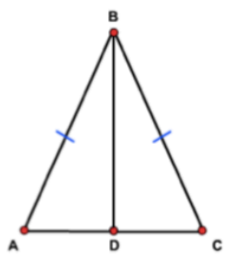

3. Consider the two triangles above. Suppose **D** is the midpoint of both $\overline{GE}$ and $\overline{FH}$. Are you given enough information to prove the triangles are congruent? If so, explain your reasoning. **[EX2, pages 3 and 4]**

   The fact that **D** is the midpoint of $\overline{GE}$ and $\overline{FH}$ tells you that **DF** = **DH** and **DE** = **DG**. $\angle FDE \cong \angle HDG$ because they are vertical angles. This gives you enough information to prove the triangles are congruent by SAS.

4. How would you complete the following congruence statement? **[EX2, page 4]**

   $\Delta FDE \cong \Delta HDG$

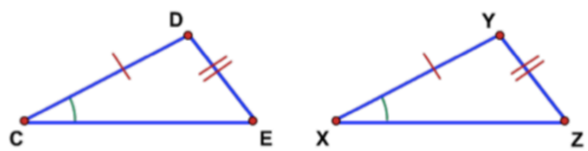
7. In this diagram, **D** is the midpoint of $\overline{AC}$ and $\overline{AB} \cong \overline{CB}$. Can you prove any triangles congruent? Explain your reasoning. **[EX2, page 9]**




*Because* **D** *is the midpoint of* $\overline{AC}$, **AD = DC**. *You are given* $\overline{AB} \cong \overline{CB}$. *That gives you two sides. But that is not enough information to prove the triangles congruent. You could use the Isosceles Triangle Theorem to say* $\angle A \cong \angle C$. *This would give you two pairs of sides and one pair of angles. You could then say* $\Delta ABD \cong \Delta CBD$ *by SAS.*

*Another approach would be to use the fact that* $\overline{DB}$ *is a shared side and is congruent to itself. Then you could say* $\Delta ABD \cong \Delta CBD$ *by SSS. Sometimes there is more than one way to prove two triangles congruent.*

8. Consider the triangles in this diagram. Suppose you know that **CD = XY**, **DE = YZ**, and $\angle C \cong \angle X$. Is this enough information to prove the triangles congruent? Explain your reasoning. **[EX2, pages 10 and 11]**



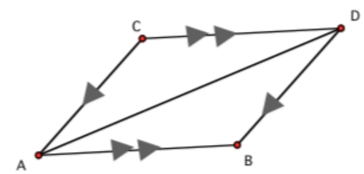

*With the given congruences, this is an example of SSA. Remember, SSA is* **not** *a valid congruent triangle postulate. Therefore, you cannot prove the two triangles congruent with the given information.*



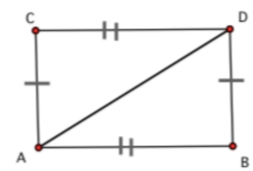



9. **REINFORCE** For each triangle pair below, decide if you can determine a triangle congruence from the given information. If so, write the triangle congruence statement and what postulate you can use. If not, explain why no congruence can be determined.

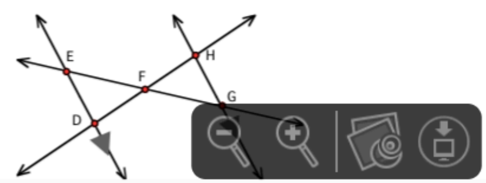
a. $\Delta ABD \cong \Delta DCA$ *by ASA. (*$\angle CAD \cong \angle BDA$ *and* $\angle CDA \cong \angle BAD$ *by the Alternate Interior Angles Postulate;* $\overline{AD} \cong \overline{AD}$ *by the Reflexive Property.)*




b. $\Delta ABD \cong \Delta DCA$ *by SSS. (Two of the sides are given congruent;* $\overline{AD} \cong \overline{AD}$ *by the Reflexive Property.)*



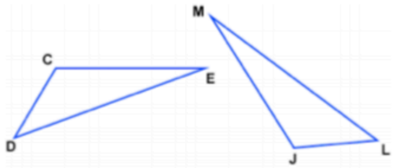

c. *No congruence can be determined. The parallel lines and the vertical lines give you AAA, which is not a triangle congruence postulate.*



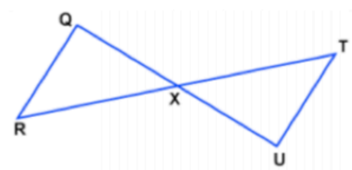
10. **REINFORCE** Use the given information, and other theorems and postulates you have learned, to decide if each pair of triangles can be proved congruent. If so, write the congruence statement and the triangle congruence postulate. You may want to mark the diagrams to show which parts are congruent.

a. Given: $\overline{CD} \cong \overline{JL}$, $\overline{CE} \cong \overline{JM}$, $\overline{DE} \cong \overline{LM}$

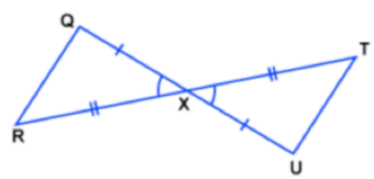
$\Delta CDE \cong \Delta JLM$ by SSS




b. Given: $\overline{QU}$ and $\overline{RT}$ bisect each other.



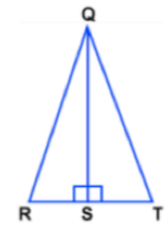

**ΔQXR ≅ ΔUXT** by SAS. Students should mark the segments congruent from the given information: $\overline{QU}$ and $\overline{RT}$ bisect each other. Students should also recognize the vertical angles in the sketch and mark them as congruent.







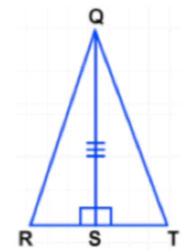

11. **REINFORCE** Prove that the following pairs of right triangles are congruent.




a. List the corresponding parts of each right triangle that must be congruent in order for **ΔQRS ≅ ΔQTS** by HL.

$\overline{QR} \cong \overline{QT}$ and $\overline{QS} \cong \overline{QS}$ or $\overline{QR} \cong \overline{QT}$ and $\overline{RS} \cong \overline{TS}$

b. If the two right triangles are marked as indicated, what additional information is necessary in order to prove that **ΔQRS ≅ ΔQTS** by ASA?




$\angle RQS \cong \angle TQS$

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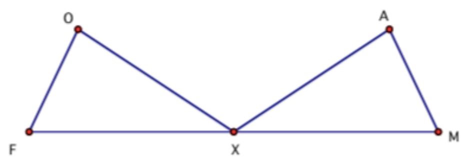
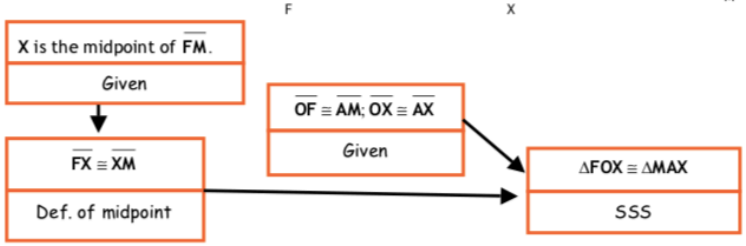
1. What are the four shortcuts you can use to prove two triangles congruent? **[EX3, page 1]**

   SSS, SAS, ASA, and SAA

2. **REINFORCE** In the following proof, the statements have been filled in for you. Write the reason in the blank below each statement.

   Given: X is the midpoint of $\overline{FM}$;
   $\overline{OF} \cong \overline{AM}$; $\overline{OX} \cong \overline{AX}$.
   Prove: $\Delta FOX \cong \Delta MAX$

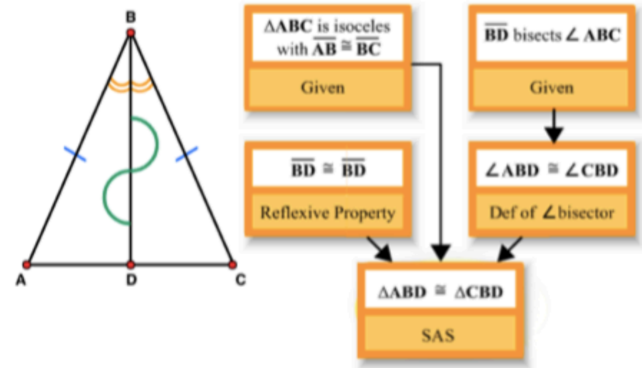




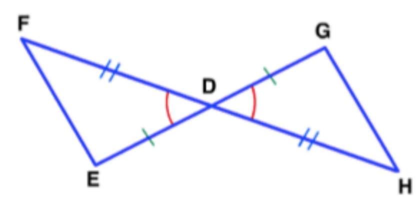



3. Complete the following proof by filling in the blanks in the flow chart. **[EX3, page 1]**




4. Write a proof of ΔFDE ≅ ΔHDG, including all the statements and reasons. **[EX3, pages 2 and 3]**

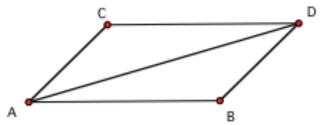
   Given: D is the midpoint of both $\overline{GE}$ and $\overline{FH}$.
   Prove: △FDE ≅ △HDG




| Statements | Reasons |
|---|---|
| 1. D is the midpoint of both $\overline{GE}$ and $\overline{FH}$. | 1. Given |
| 2. $\overline{FD} \cong \overline{DH}$ and $\overline{ED} \cong \overline{DG}$ | 2. Definition of midpoint |
| 3. ∠FDE ≅ ∠HDG | 3. Vertical angles are congruent. |
| 4. △FDE ≅ △HDG | 4. SAS |

6. **Reinforce** Write a proof of $\Delta ABD \cong \Delta DCA$. You may write a paragraph proof, a flow-chart proof, or a two-column proof.

Given: $\overline{AC} \parallel \overline{BD}$; $\overline{CD} \parallel \overline{AB}$
Prove: $\Delta ABD \cong \Delta DCA$



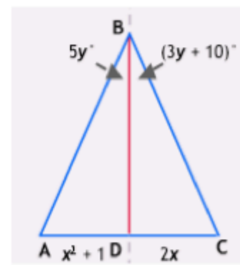

| Statements | Reasons |
|---|---|
| 1. $\overline{AC} \parallel \overline{BD}$; $\overline{CD} \parallel \overline{AB}$ | 1. Given |
| 2. $\angle CAD \cong \angle BDA$; $\angle CDA \cong \angle BAD$ | 2. Alternate Interior Angles Postulate |
| 3. $\overline{AD} \cong \overline{AD}$ | 3. Reflexive Property of Congruence |
| 4. $\Delta ABD \cong \Delta DCA$ | 4. ASA |





4/22/20

1. **Review** Suppose $\Delta ABC$ is isosceles with $AB = BC$ and altitude $\overline{BD}$. Find the length of $AC$ and $m\angle ABC$.

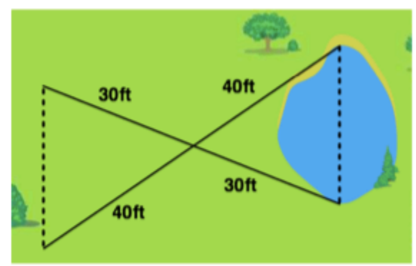



$x^2 + 1 = 2x$
$x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
$x = 1$
$AC = x^2 + 1 + 2x$
$AC = 1 + 1 + 2$
$AC = 4$ units

$5y = 3y + 10$
$2y = 10$
$y = 5$
$m\angle ABC = (5y + 3y + 10)°$
$m\angle ABC = (5(5) + 3(5) + 10)°$
$m\angle ABC = 50°$

*Note to teacher: This problem requires students to use the isosceles triangle conjectures they made in* ***Properties of a triangle.*** *Students will prove these conjectures in this topic.*

2. In order to find the distance across a pond, a surveyor helps a park ranger measure the following distances to create two triangles. How can the park ranger be sure that the two triangles are congruent? **[OV, page 1]**

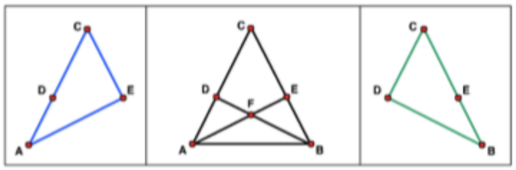



The surveyor and park ranger have measured the two pairs of sides and found that they are the same length, so each pair of sides is congruent. The included angles are congruent because they are vertical angles. Therefore, the park ranger can say that the two triangles are congruent by SAS.

3. What does CPCTC stand for? **[OV, page 2]**

   *Corresponding Parts of Congruent Triangles are Congruent.*

4. In the box to the left draw ΔCEA, and in the box to the right draw ΔCDB. **[OV, page 3]**

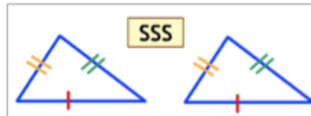
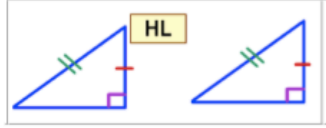








6. Fill in the blanks in the table with the abbreviations of the congruent triangle postulates. **[OV, page 6]**

| | |
|---|---|
| SSS  | If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. |
| SAS  | If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. |
| ASA  | If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. |
| SAA  | If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of another triangle, then the triangles are congruent. |
| HL  | If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the triangles are congruent. |

4/23/20:

5. **REINFORCE** Suppose you want to prove $\overline{AD} \cong \overline{DC}$. These two segments are parts of which two triangles?


These segments are parts of $\Delta ABD$ and $\Delta CBD$.

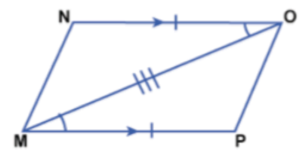




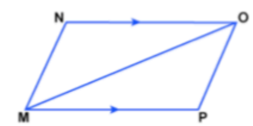

6. **REINFORCE** Given: $\overline{NO} \cong \overline{MP}$, $\overline{NO}$ P $\overline{MP}$
Prove: $\angle N \cong \angle P$

a. To complete this proof, first mark all the congruent parts on the diagram. Based on your markings, which triangles are congruent, and why? How can you use these triangles to prove $\angle N \cong \angle P$?

Mark $\overline{NO} \cong \overline{MP}$ from the given. Mark $\angle NOM \cong \angle PMO$ because they are alternate interior angles from parallel lines cut by a transversal. Finally, mark $\overline{MO} \cong \overline{MO}$ since both triangles share this segment. This makes $\Delta NOM \cong \Delta PMO$ by SAS. This means that $\angle N \cong \angle P$ by CPCTC.




b. Complete the proof by filling in the blanks.




Given: $\overline{NO} \cong \overline{MP}$, $\overline{NO}$ P $\overline{MP}$
Prove: $\angle N \cong \angle P$

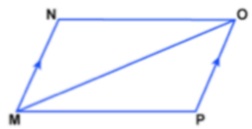
| Statements | Reasons |
|---|---|
| 1. $\overline{NO}$ P $\overline{MP}$ | 1. Given |
| 2. $\angle NOM \cong \angle PMO$ | 2. If parallel lines are cut by a transversal, then alt. int. angles are congruent. |
| 3. $\overline{NO} \cong \overline{MP}$ | 3. Given |
| 4. $\overline{MO} \cong \overline{MO}$ | 4. Reflexive property of congruence |
| 5. $\Delta NOM \cong \Delta PMO$ | 5. SAS |
| 6. $\angle N \cong \angle P$ | 6. CPCTC |

7. **Reinforce** Complete the proof by choosing statements from the list below and filling in the steps of the proof in the correct order.

**Statements:**

| | | | | | | |
|---|---|---|---|---|---|---|
| $\overline{MO} \cong \overline{MO}$ | $\angle NOM \cong \angle PMO$ | $\overline{NO}$ P $\overline{MP}$ | $\angle NMO \cong \angle POM$ | $\angle N \cong \angle P$ | $\overline{NM}$ P $\overline{OP}$ | $\Delta NOM \cong \Delta PMO$ |

Given: $\overline{NM}$ P $\overline{OP}$, $\angle N \cong \angle P$
Prove: $\overline{NO}$ P $\overline{MP}$



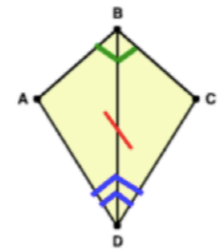

| Statement | Reasons |
|---|---|
| 1. $\overline{NM}$ P $\overline{OP}$ | 1. Given |
| 2. $\angle NMO \cong \angle POM$ | 2. If parallel lines are cut by a transversal, then alternate interior angles are congruent. |
| 3. $\angle N \cong \angle P$ | 3. Given |
| 4. $\overline{MO} \cong \overline{MO}$ | 4. Reflexive property of congruence |
| 5. $\Delta NOM \cong \Delta PMO$ | 5. AAS |
| 6. $\angle NOM \cong \angle PMO$ | 6. CPCTC |
| 7. $\overline{NO}$ P $\overline{MP}$ | 7. If the alternate interior angles are congruent, then the lines are parallel. |





8. Mark the congruent parts of the triangles and complete the proof. **[EX1, pages 4-5]**

| | | | | |
|---|---|---|---|---|
| SAS | Reflexive | $\Delta ABD \cong \Delta CBD$ | Def ∠ bis. | ASA |
| $\angle A \cong \angle C$ | $\Delta ABD \cong \Delta DBC$ | CPCTC | Given | Symmetric |

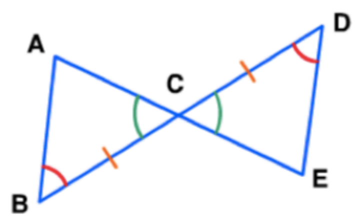



| Statements | Reasons |
|---|---|
| 1. $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$ | 1. Given |
| 2. $\angle ABD \cong \angle CBD$; $\angle ADB \cong \angle BDC$ | 2. Def ∠ bis. |
| 3. $\overline{BD} \cong \overline{BD}$ | 3. Reflexive property |
| 4. $\triangle ABD \cong \triangle CBD$ | 4. ASA |
| 5. $\angle A \cong \angle C$ | 5. CPCTC |

9. Mark the congruent parts of the triangles and complete the proof. **[EX1, pages 6-7]**

| | | | | |
|---|---|---|---|---|
| Given | ASA Postulate | $\Delta BCA \cong \Delta DCE$ | CPCTC | $\angle ACB \cong \angle ECD$ |
| $\overline{AE}$ bisects $\overline{BD}$ | $\overline{BC} \cong \overline{CD}$ | $\overline{AC} \cong \overline{CE}$ | $\overline{DB}$ bisects $\overline{AE}$ | |

**Given:** $\angle B \cong \angle D$; $\overline{AE}$ bisects $\overline{BD}$
**Prove:** $\overline{DB}$ bisects $\overline{AE}$

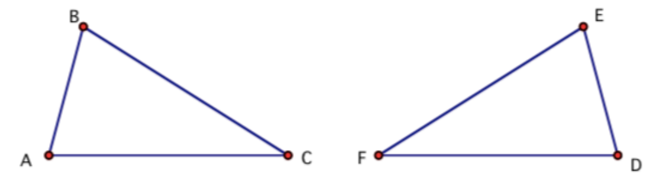



| Statements | Reasons |
|---|---|
| $\angle B \cong \angle D$ | Given |
| $\overline{AE}$ bisects $\overline{BD}$ | Given |
| $\overline{BC} \cong \overline{CD}$ | Def. of segment bisector |
| $\angle ACB \cong \angle ECD$ | Vertical angles are congruent. |
| $\triangle BCA \cong \triangle DCE$ | ASA Postulate |
| $\overline{AC} \cong \overline{CE}$ | CPCTC |
| $\overline{DB}$ bisects $\overline{AE}$ | Def. of segment bisector |





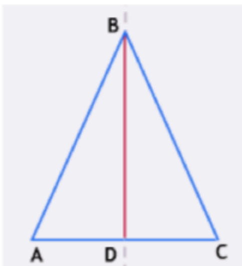
10. **Reinforce** Given $\Delta ABC \cong \Delta DEF$, name all of the corresponding parts you could prove congruent using CPCTC.




Using *CPCTC* (the definition of congruent triangles):
$m\angle A \cong m\angle D$; $m\angle B \cong m\angle E$; $m\angle C \cong m\angle F$.
$\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$; $\overline{AC} \cong \overline{DF}$.

4/24/20:

1. List the isosceles triangles conjectures you made in the topic **Properties of a Triangle. [EX2, page 1]**




The base angles of an isosceles triangle are congruent.
$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.
$\overline{BD}$ bisects $\angle ABC$.
$\Delta ABD \cong \Delta CBD$.

2. **Reinforce** If the vertex angle of an isosceles triangle has a measure of 50°, what are the measures of the two base angles? Explain your solution.

Because the triangle is isosceles, the two base angles are congruent and will have the same measure. Let $x$ be the measure of each base angle. Use the Triangle Sum Theorem to set up an equation.

$$x + x + 50 = 180$$
$$2x = 130$$
$$x = 65$$

Therefore, each base angle measures 65°.





7. **Reinforce** Consider the isosceles triangle in this diagram. How can we use this diagram to prove the Isosceles Triangle Theorem without drawing an auxiliary line? Parts a and b below will help you answer this question.


a. Decide if each of the following triangle congruence statements is true or false. Explain your reasoning.

True $\Delta ABC \cong \Delta ABC$
By the Reflexive Property, a triangle is congruent to itself.

True $\Delta ABC \cong \Delta CBA$
From the triangle congruence statement, the corresponding sides are as follows: $\overline{AB}$ and $\overline{CB}$; $\overline{BC}$ and $\overline{BA}$; $\overline{AC}$ and $\overline{CA}$. The order in which we name a segment does not matter. $\overline{AC}$ and $\overline{CA}$ are the same segment and congruent by the Reflexive Property.
$\overline{AB} \cong \overline{CB}$ and $\overline{BC} \cong \overline{BA}$ because these are the congruent sides of the isosceles triangle. Therefore, the triangle congruence statement is true by SSS.

False $\Delta ABC \cong \Delta BAC$
From the triangle congruence statement, the corresponding sides $\overline{BC}$ and $\overline{CA}$ would need to be congruent. However, these are not congruent, based on the diagram. Therefore, in the order written, this is not a true congruence statement.

b. Which one of the true triangle congruency statements above can be used to prove the Isosceles Triangle Theorem? Explain your reasoning.

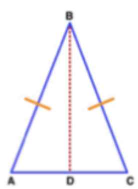
$\Delta ABC \cong \Delta CBA$ can be used to prove the Isosceles Triangle Theorem because, given the correspondence as written, $\angle A \cong \angle C$ by **CPCTC**.

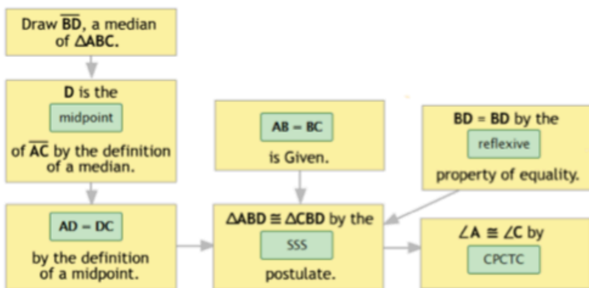
6. Complete this proof of the Isosceles Triangle Theorem. **[EX2, page 3]**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| AD = DC | CPCTC | reflexive | AB = BC | symmetric | SSA | midpoint | SSS |

**Given**: AB = BC

**Prove** ∠A ≅ ∠C





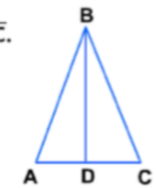



8. Parts a and b below guide the proofs for the remaining isosceles triangle conjectures.

a. Complete the proof of the third conjecture: $\overline{BD}$ bisects ∠ABC. **[EX2, page 5]**

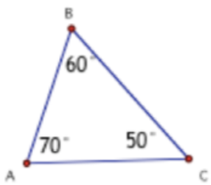
| | | | | | | |
|---|---|---|---|---|---|---|
| CPCTC | ΔDAB ≅ ΔDBC | $\overline{BD}$ bisects ∠ABC | Definition of midpoint | ΔABD ≅ ΔCBD | Given | SSS |

**Given:** AB = BC and $\overline{BD}$ is the median of $\overline{AC}$.
**Prove:** $\overline{BD}$ bisects ∠ABC.




| Statements | Reasons |
|---|---|
| 1. AB = BC | 1. Given |
| 2. $\overline{BD}$ is the median of $\overline{AC}$. | 2. Given |
| 3. △ABD ≅ △CBD | 3. Median of isos. △ forms two ≅ △s. |
| 4. ∠ABD ≅ ∠CBD | 4. CPCTC |
| 5. $\overline{BD}$ bisects ∠ABC | 5. Def. of ∠ bisector |

14. **Reinforce** Given the triangle below with the angle measures shown, rank the side lengths in order from smallest to greatest.




$\underline{AB} < \underline{AC} < \underline{BC}$