Grade 6
Family Resource Bundle

Grade 6

ANSWER KEY Text #1 Noticing Mistakes Boosts Learning

by Alison Pearce Stevens 2017

1. RI.KID.2

PART A: Which of the following describes the central idea of the text?

A. Accepting and learning from your mistakes helps you improve when you try again.
B. Students have been taught to avoid mistakes rather than accept them.
C. Making mistakes shows that you're someone who's not afraid to take risks.
D. People with fixed mindset don't learn from their mistakes because they don't make them often.

2. RI.KID.1

PART B: Which detail from the text best supports the answer to Part A?

A. "Mistakes get a bad rap. People often brush them aside by saying, 'I'll do better next time.'" (Paragraph 1)
B. "It is when most kids are beginning school. How well they do in school can be related to their mindset about learning and intelligence." (Paragraph 3)
C. "Students who have a 'fixed' mindset tend to believe that they are born with a certain level of intelligence. They don't believe it can ever change." (Paragraph 4)
D. "Children with growth mindsets were also better at bouncing back after their mistakes. 'They were more likely to get the next trial right'" (Paragraph 8)

3. RI.CS.6

Which statement describes the author's main purpose in the text?

A. to encourage readers to make as many mistakes as they can
B. to show how adults hurt students' intelligence by discouraging mistakes
C. to provide evidence for how mistakes can help you learn
D. to help readers determine if they have a growth mindset or fixed mindset

4. RI.KID.3

Which statement describes the relationship between fixed mindset and growth mindset?

A. Fixed mindset and growth mindset describes how our brains solve challenging problems.
B. Fixed mindset and growth mindset show how a person views their own intelligence.
C. Fixed mindset shows that someone has learned as much as they can while growth mindset shows they have more to learn.
D. Fixed mindset is the reluctance to learn any more while growth mindset is a person's desire to improve themselves.

5. RI.CS.5

How does the author's discussion of Schroder's study contribute to the development of ideas about how children react to mistakes?

1. **Answers will vary; students should discuss how Schroder's study supports the idea that children who are willing to accept their mistakes, rather than ignore them, are more likely to learn from them and improve. Schroder determined whether the participants had growth mindset or fixed mindset and then measured the activity in their brain when they made mistakes. The study showed that "the brains of children with a growth mindset showed much more activity. What's more, a larger network of areas responded" (Paragraph 7). Additionally, the children's brains remained active longer: "This shows that these brains were paying attention to mistakes" (Paragraph 7). While children with growth mindset appeared to be more engaged in the test, they also had more impressive results than the children with fixed mindsets. Schroder found that "'they [the children with growth mindset] were more likely to get the next trial right'" (Paragraph 8). Overall, Schroder's study helps prove that children who pay attention to their mistakes are more likely to learn and improve.**

ANSWER KEY Text #2 The Crow and the Pitcher

by Aesop 620-560 BCE

1. RL.CS.4

PART A: What does the word "spell" mean as it is used in paragraph 1?

A. a saying with magical powers
B. a type of weather
C. a period of time
D. a land needing water

2. RL.KID.1

PART B: Which phrase from paragraph 1 provides the best support for your answer to Part A?

A. "a thirsty crow"
B. "when the birds could find very little"
C. "a little water in it"
D. "found a pitcher"

3. RL.KID.3

What does the information in paragraph 2 reveal about the crow?

A. He is not able to solve a problem.
B. He is resourceful and clever.
C. He is extremely strong.
D. He knows when to ask for help.

4. RL.CS.5

How does paragraph 2 contribute to the story's resolution?

A. After not being able to find anything to drink, the crow decides to ask for help.
B. After having lots of water, the crow now can't find any.
C. After struggling to get the water from the pitcher, the crow finds a solution.
D. After not being able to get water from the pitcher, the crow decides to look in a new place.

5. RL.KID.2

Explain the theme or lesson of the story. Use evidence from the story to support your answer.

1. **Answers will vary. Students should summarize what they think the main lesson is in the story. For example, students might discuss that the crow was close to dying of thirst (Paragraph 1), but he came up with an idea to get water from the pitcher (Paragraph 2), and he was successful in getting the water from the pitcher (Paragraph 2). Students should then go on to explain these pieces of evidence support one of three themes: "it is important to be solutions oriented/stay positive," "you'll do well if you are clever in a difficult situation," or "never give up."**

Related Media Links and Descriptions

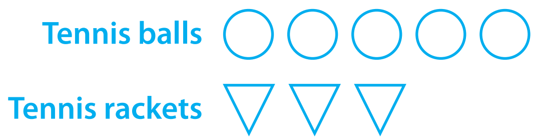
Related Media #1: Mindsets: Fixed Versus Growth
Show this video to students to provide them with additional information about fixed mindset and growth mindset. 2:19

Related Media #2: Sesame Street- The Crow and the Pitcher
This animated video is a quick version of Aesop's classic tale. 1:12

Understanding Ratio Concepts

➤ **Complete each problem about ratio relationships.**

1. Ms. Omar runs the school tennis club. She has a bin of tennis balls and rackets. For every 5 tennis balls in the bin, there are 3 tennis rackets. Draw a model to show the ratio of tennis balls to tennis rackets.



Write the following ratios.

tennis balls to tennis rackets **5 : 3 or 5 to 3**

tennis balls to total pieces of tennis equipment **5 : 8 or 5 to 8**

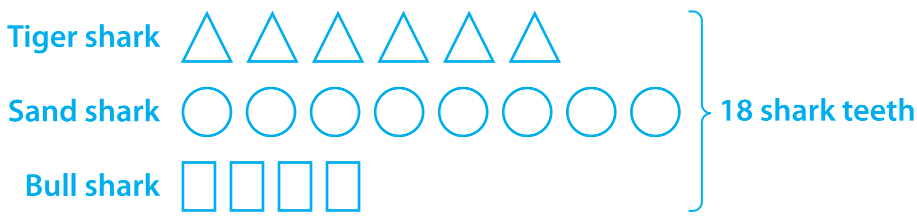
2. Christian has a collection of 18 shark teeth. He identified them as 6 tiger shark teeth, 8 sand shark teeth, and the rest as bull shark teeth.

What does the ratio 6 : 8 represent in this situation?

Possible answer: The ratio 6 : 8 is the ratio of the number of tiger shark teeth to the number of sand shark teeth.

What does the ratio 4 : 18 represent in this situation? Explain your reasoning. Include a model in your explanation.

Possible answer: 6 + 8 = 14 and 18 − 14 = 4, so there are 4 bull shark teeth and a total of 18 shark teeth. So, 4 : 18 is the ratio of bull shark teeth to the total number of shark teeth.



3. How are part-to-part ratios different from part-to-whole ratios?

Possible answer: Part-to-part ratios show the relationship between two separate groups that are part of a whole. Part-to-whole ratios show the relationship between one or more parts and the total number of items that make up the whole.

Using Equivalent Ratios

Solve each problem.

1. Josie is training for a race. The ratio of the number of minutes she runs to the number of miles she runs is 24 to 3. She plans to run 10 miles. How many minutes will it take her?

   80 minutes

2. A chef planning for a large banquet thinks that 2 out of every 5 dinner guests will order his soup appetizer. He expects 800 guests at the banquet. Use equivalent ratios to estimate how many cups of soup he should prepare.

   320 cups of soup

3. Fred is making a fruit salad. The ratio of cups of peaches to cups of cherries is 2 to 3. How many cups of peaches will Fred need to make 60 cups of fruit salad?

   24 cups of peaches

4. A community garden center hosts a plant giveaway every spring to help community members start their gardens. Last year, the giveaway supported 50 families by giving away 150 plants. Based on this ratio, how many plants will the center give away this year in order to support 65 families?

   195 plants

5. The first week of January, there are 49 dogs and 28 cats in an animal shelter. Throughout the month, the ratio of dogs to cats remains the same. The last week of January, there are 20 cats in the shelter. How many dogs are there?

6. A wedding planner uses 72 ivy stems for 18 centerpieces. When she arrives at the venue, she realizes she will only need 16 centerpieces. How many ivy stems should she use so that the ratio of ivy stems to centerpieces stays the same?

Find Equivalent Ratios

● Check Understanding

Possible answer:

Red	**6**	12	18	24	30
Blue	**5**	10	15	20	25

Possible explanation: The ratios are equivalent to 6 : 5 because I multiplied both numbers in the ratio by the same number to find each equivalent ratio.

ACTIVITY ANSWERS

The order of ratios in each table may vary.

2	4	6	8	10
1	2	3	4	5

2	4	6	8	10
3	6	9	12	15

1	2	3	4	5
3	6	9	12	15

1	2	3	4	5
4	8	12	16	20

5	10	15	20	25
1	2	3	4	5

4	8	12	16	20
3	6	9	12	15

●● Check Understanding

Possible answer:

Red	6	**12**	18	24	30
Blue	5	**10**	15	20	25

Possible explanation: The ratios are equivalent to 12 : 10 because I multiplied or divided both numbers in the ratio by the same number to find each equivalent ratio.

ACTIVITY ANSWERS

The order of ratios in each table may vary.

2	**4**	6	8	10
1	**2**	3	4	5

2	**4**	6	8	10
3	**6**	9	12	15

1	2	**3**	4	5
3	6	**9**	12	15

1	2	**3**	4	5
4	8	**12**	16	20

5	**10**	15	20	25
1	**2**	3	4	5

Find Equivalent Ratios *continued*

4	8	**12**	16	20
3	6	**9**	12	15

The following Ratio Cards are not used:
15 : 5, 5 : 6, 4 : 10, 3 : 1, 4 : 1, 9 : 3

●●● Check Understanding

There are 66 red marbles in the large bag and 18 red marbles in the small bag.

Possible work:

		÷ 4	× 11
Red	24	6	**66**
Blue	20	5	55

		÷ 4	× 3
Red	24	6	**18**
Blue	20	5	15

ACTIVITY ANSWERS

The order of ratios in each table may vary.

2	**4**	6	8	16
3	**6**	9	12	24

5	10	**15**	20	40
3	6	**9**	12	24

3	6	9	**15**	24
5	10	15	**25**	40

4	8	**12**	16	32
3	6	**9**	12	24

3	9	12	**18**	24
4	12	16	**24**	32

3	6	9	**12**	24
2	4	6	**8**	16

The following Ratio Cards are not used:
3 : 1, 4 : 1, 9 : 3, 15 : 5, 5 : 6, 4 : 10, 15 : 11, 15 : 12, 4 : 5, 3 : 8, 10 : 4, 12 : 2

Understanding Rate Concepts

1 It takes Maya 30 minutes to solve 5 logic puzzles, and it takes Amy 28 minutes to solve 4 logic puzzles. Use models to show the rate at which each student solves the puzzles, in minutes per puzzle.

Possible answer:

Maya

Minutes	Number of puzzles
30	5
6	1

Amy

Minutes	Number of puzzles
28	4
7	1

If Maya and Amy had the same number of puzzles to solve, who would finish first? Explain.

Maya will finish first. Possible explanation: Maya takes 6 minutes per puzzle. Amy takes 7 minutes per puzzle.

2 A garden hose supplies 36 gallons of water in 3 minutes. Use a table of equivalent ratios to show the garden hose's water flow in *gallons per minute* and *minutes per gallon*.

Possible work:

Gallons	36	12	1
Minutes	3	1	$\frac{1}{12}$

12 gallons per minute; $\frac{1}{12}$ minute per gallon

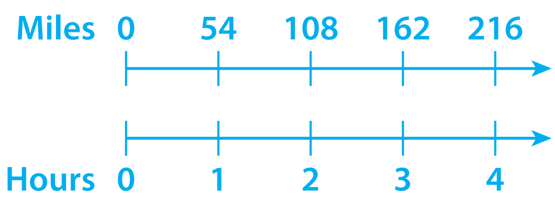
How many gallons of water does the hose supply in 10 minutes? Explain.

Possible answer: You multiply the rate in gallons per minute by 10 minutes. $10 \times 12 = 120$ gallons of water in 10 minutes.

Understanding Rate Concepts *continued*

3 Max travels to see his brother's family by car. He drives 216 miles in 4 hours. What is his rate in miles per hour? Use a double number line to show your work.

54 miles per hour; Possible work:



Suppose he makes two stops of 10 minutes each during his journey. Will he be able to reach the town in 4 hours if he keeps the speed the same?

No; Possible explanation: If he makes two 10-minute stops, he will have to travel the distance in 3 hours and 40 minutes, which means that he will not be able to reach the town in 4 hours without driving faster.

Using Unit Rates to Find Equivalent Ratios

Solve each problem. Show your work.

1. Rachel mows 5 lawns in 8 hours. At this rate, how many lawns can she mow in 40 hours?

   25 lawns; Possible work: Since $5 \div 8 = \frac{5}{8}$, Rachel mows $\frac{5}{8}$ lawns per hour: $40 \times \frac{5}{8} = 25$. So, Rachel mows 25 lawns in 40 hours.

2. A contractor charges $1,200 for 100 square feet of roofing installed. At this rate, how much does it cost to have 1,100 square feet installed?

   $13,200; Possible work: Since $1{,}200 \div 100 = 12$, the roof installation costs $12 per square foot: $1{,}100 \times 12 = 13{,}200$. So, it costs $13,200 to install 1,100 square feet.

3. It takes Jill 2 hours to run 14.5 miles. At this rate, how far could she run in 3 hours?

   21.75 miles; Possible work: Since $14.5 \div 2 = 7.25$, Jill runs about 7.25 miles per hour: $3 \times 7.25 = 21.75$. So, Jill could run 21.75 miles in 3 hours.

4. Bobby catches 8 passes in 3 football games. At this rate, how many passes does he catch in 15 games?

   40 balls; Possible work: Since $8 \div 3 = \frac{8}{3}$, Bobby catches $\frac{8}{3}$ balls per game: $15 \times \frac{8}{3} = 40$. So, Bobby catches 40 balls in 15 games.

5. Five boxes of crackers cost $9. At this rate, how much do 20 boxes cost?

   $36; Possible work: Since $9 \div 5 = \frac{9}{5} = 1.80$, the crackers cost $1.80 per box: $20 \times 1.80 = 36$. So, 20 boxes cost $36.00.

6. It takes a jet 2 hours to fly 1,100 miles. At this rate, how far does it fly in 8 hours?

   4,400 miles; Possible work: Since $1{,}100 \div 2 = 550$, the jet flies 550 miles per hour: $550 \times 8 = 4{,}400$. So, it flies 4,400 miles in 8 hours.

Using Unit Rates to Find Equivalent Ratios *continued*

7 It takes Dan 32 minutes to complete 2 pages of math homework. At this rate, how many pages does he complete in 200 minutes?

12.5 pages; Possible work: Since $2 \div 32 = 0.0625$, Dan completes 0.0625 page per minute: $0.0625 \times 200 = 12.5$. So, he completes 12.5 pages in 200 minutes.

8 Kendra gets a paycheck of \$300 after 5 days of work. At this rate, how much does she get paid for working 24 days?

\$1,440; Possible work: Since $300 \div 5 = 60$, Kendra gets paid \$60 per day: $60 \times 24 = 1{,}440$. So, she gets paid \$1,440 for working 24 days.

9 Tim installs 50 square feet of his floor in 45 minutes. At this rate, how long does it take him to install 495 square feet?

550 minutes; Possible work: Since $50 \div 45 = \frac{10}{9}$, Tim installs $\frac{10}{9}$ square feet per minute: $495 \times \frac{10}{9} = 550$. So, it takes him 550 minutes to install 495 square feet.

10 Taylin buys 5 ounces of tea leaves for \$2.35. At this rate, how much money does she need to buy 12 ounces of tea leaves?

\$5.64; Possible work: Since $2.35 \div 5 = 0.47$, tea leaves cost \$0.47 per ounce: $0.47 \times 12 = 5.64$. So, she needs \$5.64 to buy 12 ounces.

11 In problem 10, how would your work be different if you were asked how many ounces of tea leaves Taylin could buy with \$10?

I would find the unit rate in terms of ounces per dollar rather than dollars per ounce and then multiply by \$10 to find the number of ounces Taylin could buy with that amount.

Using Unit Rates to Compare Ratios

Solve each problem. Show your work.

1. Shawn sells 36 vehicles in 4 weeks. Brett sells 56 vehicles in 7 weeks. Who sells more vehicles per week?

   Shawn; Possible work: Shawn: $\frac{36}{4} = 9$ vehicles per week; Brett: $\frac{56}{7} = 8$ vehicles per week; $9 > 8$

2. The table shows the gas mileage of two vehicles. Which vehicle travels more miles per gallon?

Car	Miles	Gallons
Pickup Truck	120	8
Minivan	180	10

   Minivan; Possible work: Pickup Truck: $\frac{120}{8} = 15$; Minivan: $\frac{180}{10} = 18$; 18 mpg $>$ 15 mpg

3. Joe and Chris each have a lawn mowing business. Joe charges $40 to mow 2 acres. Chris charges $30 to mow 1.2 acres. Who charges more per acre?

   Chris; Possible work: Joe: $\frac{40}{2} = 20$; Chris: $\frac{30}{1.2} = 25$; \$25 $>$ \$20

4. The table shows the time it took two athletes to run different races. Who ran faster?

Athlete	Seconds	Meters
Ellen	28	200
Lindsay	60	400

   Ellen; Possible work: Ellen: $\frac{200}{28} \approx 7.14$ meters per second; Lindsay: $\frac{400}{60} \approx 6.67$ meters per second; $6.67 < 7.14$

Using Unit Rates to Compare Ratios *continued*

5 Branden and Pete each play running back. Branden carries the ball 75 times for 550 yards, and Pete has 42 carries for 380 yards. Who runs farther per carry?

Pete; Possible work: Branden: $\frac{550}{75} \approx 7.33$ yards per carry;
Pete: $\frac{380}{42} \approx 9.05$ yards per carry; $9.05 > 7.33$

6 The table shows the price of two cereal brands and the number of ounces per box. Which is the better price per ounce?

Cereal	Ounces	Price
Brand A	18	$2.50
Brand B	24	$3.50

Brand A; Possible work: Brand A: $\frac{2.50}{18} \approx 0.14$; Brand B: $\frac{3.50}{24} \approx 0.15$;
$\$0.14 < \$0.15$

7 Describe two different ways you could change the values in the table so that the answer to problem 6 is different.

Possible answer: I could change the price of Brand B to $3.35 or less or change the number of ounces for Brand B to 25 ounces or more.

Using Unit Rates to Convert Measurements

► **Solve each problem. Show your work.**

1. Susan has a 12-inch board for constructing a wooden chair. The directions say to use a board that is 29 centimeters long. Is her board long enough to cut? (1 inch = 2.54 centimeters)

   Yes; Possible work: 2.54 centimeters per inch: $12 \times 2.54 = 30.48$

   Her board is 30.48 centimeters long, so she has enough to cut 29 centimeters.

2. Kevin uses 84 fluid ounces of water to make an all-purpose cleaner. The directions call for 4 fluid ounces of concentrated soap for every 3 cups of water. How many fluid ounces of soap should he use? (1 cup = 8 fl oz)

   14 fluid ounces of soap; Possible work: 8 fl oz per cup: $8 \times 3 = 24$ fl oz of water

   4 fl oz of soap per 24 fl oz of water: $\frac{4}{24} = \frac{1}{6}$ fl oz of soap per fl oz of water

   $84 \times \frac{1}{6} = 14$

3. Shannon test-drives a car in Germany and drives 95 kilometers per hour. What is her speed in miles per hour? (1 kilometer ≈ 0.62 mile)

   58.9 miles per hour; Possible work: 0.62 mile per kilometer: $95 \times 0.62 = 58.9$

4. Keith works 8 hours per day for 5 days per week. Melba works 2,250 minutes each week. Who spends more time at work?

   Keith; Possible work: 60 minutes in 1 hour; $8 \times 5 = 40$ hours per week; $40 \times 60 = 2{,}400$ minutes, so Keith works 2,400 minutes each week. This is more than 2,250, so Keith spends more time at work.

Using Unit Rates to Convert Measurements *continued*

5 Jason runs 440 yards in 75 seconds. At this rate, how many minutes does it take him to run a mile? (1 mile = 1,760 yards)

5 minutes; Possible work: $\frac{1}{1{,}760}$ miles per yard, $440 \times \frac{1}{1{,}760} = \frac{1}{4}$ mile

He runs $\frac{1}{4}$ mile in 75 seconds, so it takes him $75 \times 4 = 300$ seconds to run a mile.

$\frac{1}{60}$ min per second, $300 \times \frac{1}{60} = 5$ minutes

6 Boxes of granola are on sale at a price of 2 for \$4.50. There are 12 ounces of granola in each box. What is the unit price in dollars per pound?

\$3.00 per pound; Possible work: $12 \times 2 = 24$ total ounces; 16 ounces in 1 pound; $\frac{24}{16} = 1.5$ pounds; $\frac{4.50}{1.5} = \$3.00$ per pound

7 Sam is delivering two refrigerators that each weigh 105 kilograms. There is an elevator with a weight limit of 1,000 pounds. Can he take both refrigerators on the elevator in one trip? (1 kilogram ≈ 2.2 pounds)

Yes; Possible work: 2.2 pounds per kilogram; $105 \times 2.2 = 231$; $231 \times 2 = 462$

Sam can take the refrigerators in the elevator in one trip because the combined weight of the refrigerators is only 462 pounds.

8 For every 140 feet that Kelly rides on her bicycle, the wheels turn 20 times. About how many times do the wheels turn in 5 miles? (1 mile = 5,280 feet)

about 3,771 times; Possible work: 5,280 feet per mile, $\frac{20}{140} = \frac{1}{7}$ turn per foot, $5 \times 5{,}280 = 26{,}400$ feet; $26{,}400 \times \frac{1}{7} = 3{,}771.43$ turns

Use Ratio and Rate Vocabulary

● Check Understanding

$10.50; Possible answer: First, I divide the numbers in the ratio, 7.50 : 5, to find the unit rate for dollars per pound: $7.50 \div 5 = 1.50$. Then I multiply the number of pounds by the unit rate: $7 \times 1.50 = 10.50$. So, a 7-lb bag of apples would cost $10.50.

RECORDING SHEET

I know the units need to be the same to compare the prices. First, I **convert** the length of the flannel fabric to feet. The **ratio** of feet to yards is 3 feet : 1 yard. The **rate** is 3 feet per yard. The **unit rate** is 3. I **multiply** the number of yards by the unit rate. The result is 9.

The flannel fabric has a length of 9 feet. Now, I can find the unit cost of each fabric in **dollars** per **foot**. I **divide** to find an **equivalent ratio**.

The flannel fabric costs $2 per foot. The fleece fabric costs $2.20 per foot. The flannel fabric costs **less** per foot, so it is the better buy.

●● Check Understanding

$6.40 per yard; Possible answer: First, I need to convert the length of the fabric to yards. I know the ratio of feet to yards is 3 feet : 1 yard. I divide the length in feet by 3 to find the length of the fabric in yards: $4.5 \div 3 = 1.5$. Then I divide the cost by the length in yards to find the cost per yard: $9.60 \div 1.5 = 6.40$. So, the cost per yard is $6.40.

RECORDING SHEET

I know the units need to be the same to compare the prices. First, I **convert** the length of the flannel fabric to feet. The **ratio** of feet to yards is 3 feet : 1 yard. The **rate** is 3 feet per yard. The **unit rate** is 3. I **multiply** the number of yards by the unit rate. The result is **9**.

The flannel fabric has a length of **9** feet. Now, I can find the unit cost of each fabric in **dollars** per **foot**. I **divide** to find an **equivalent ratio**.

Flannel

Price ($)	12.60	**1.40**
Length (ft)	**9**	1

Fleece

Price ($)	8.75	**1.75**
Length (ft)	**5**	1

The flannel fabric costs $**1.40** per foot. The fleece fabric costs $**1.75** per foot. The flannel fabric costs **less** per foot, so it is the better buy.

Use Ratio and Rate Vocabulary *continued*

Check Understanding

\$6.40 per yard; Possible answer: First, I have to convert the length of the fabric to yards. I know the ratio of feet to yards is 3 feet : 1 yard. So, the unit rate is 3. I divide the length in feet by the unit rate to find the length of the fabric in yards: $4.5 \div 3 = 1.5$. Then I divide the cost by the length in yards to find the cost per yard: $\$9.60 \div 1.5 = \$6.40$. The cost per yard is \$6.40.

RECORDING SHEET

I know the units need to be the same to compare the prices. First, I **convert** the number of gallons Bottle A holds to fluid ounces. The **ratio** of fluid ounces to gallons is 128 fluid ounces : 1 gallon. The **rate** is **128** fluid ounces per gallon. The **unit rate** is 128. I **multiply** the number of gallons by the **unit rate**. The result is **64**.

Bottle A holds **64** fluid ounces. Now, I can find the cost of each bottle in **dollars** per **fluid ounce**. I **divide** to find an **equivalent ratio**.

Price (\$)	5.12	**0.08**
Fluid Ounces	**64**	1

Price (\$)	3.64	**0.07**
Fluid Ounces	**52**	1

The 0.5-gallon bottle costs \$**0.08** per fluid ounce.

The 52-fluid ounce bottle costs \$**0.07** per fluid ounce.

The 52-fluid ounce bottle costs **less** per fluid ounce, so it is the better buy.

6.RP Games at Recess

Alignments to Content Standards: 6.RP.A.1

Task

The students in Mr. Hill's class played games at recess.

6 boys played soccer
4 girls played soccer
2 boys jumped rope
8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games.

Mika said,

"Four more girls jumped rope than played soccer."

Chaska said,

"For every girl that played soccer, two girls jumped rope."

Mr. Hill said, "Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio."

a. Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.

b. Compare the number of boys who played soccer and jumped rope using a ratio. Write your answer as a sentence as Chaska did.

c. Compare the number of girls who played soccer to the number of boys who played soccer using a ratio. Write your answer as a sentence as Chaska did.

IM Commentary

In a classroom where the expectation is built in that answers to problems in context will be written as complete sentences and numerical values from a context will always be written with the appropriate units, the task may not need to explicitly model and request it as these questions do.

While students need to be able to write sentences describing ratio relationships, they also need to see and use the appropriate symbolic notation for ratios. If this is used as a teaching problem, the teacher could ask for the sentences as shown, and then segue into teaching the notation. It is a good idea to ask students to write it both ways (as shown in the solution) at some point as well.

Edit this solution

Solution

a. Four more boys played soccer than jumped rope.

b. For every three boys that played soccer, one boy jumped rope. Therefore the ratio of the number of boys that played soccer to the number of boys that jumped rope is 3:1 (or "three to one").

c. For every two girls that played soccer, three boys played soccer. Therefore the ratio of the number of girls that played soccer to the number of boys that played soccer is 2:3 (or "two to three").

The Escalator, Assessment Variation

Sample task from achievethecore.org
By Illustrative Mathematics and Student Achievement Partners

GRADE LEVEL Sixth

IN THE STANDARDS 6.RP.A.1, 6.RP.A.2

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Provides a simply stated, yet mathematically rich task.
- Requires students to understand the concept of a ratio (6.RP.A.1) and a rate (6.RP.A.2).
- Builds understanding of ratios through the use of precise mathematical language (e.g., every, per) (MP6).

In the classroom:

- Enables students to consider multiple correct descriptions of the same ratio by using "Choose all that apply."
- Orders answer choices intentionally, placing a similar – but incorrect – choice (c) after a correct one (a).
- Gives students the opportunity to thoughtfully select the method they will use to solve the task (e.g., table of equivalent ratios, plotting points in the coordinate plane, double number line diagrams, equations) (MP5).

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides opportunities to support students in connecting mathematical language with mathematical representations.
- Prompts teachers to write essential ideas/concepts/language on the board as a reference for students.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of grade 6
Coherence	Provides foundational work for learning about proportional relationships in grade 7 (see Molly's Run)
Rigor[3]	Conceptual Understanding: primary in this task
	Procedural Skill and Fluency: secondary in this task
	Application: secondary in this task

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Six.
[3]Tasks will often target only one aspect of Rigor.

INSTRUCTIONAL ROUTINE

The steps in this routine are adapted from the Principles for the Design of Mathematics Curricula: Promoting Language and Content Development.

Engage students in the Collect and Display Mathematical Language Routine as a way to capture the language they use when thinking about this task. This will provide a stable, collective reference for students to refer to, build on, or make connections to while working on future tasks. This collection can be used as a model and then revised and updated as more content is learned.

As students are working to find the correct statements in this task, circulate and listen to students talk. Record important words and phrases used along with diagrams. Add these to a visual display to use during the whole-class discussion of the task. As this recording is shared, students can clarify how and why they used these words or diagrams. Ask "Which of these help our communication to be more precise?" Listen for words directly relating to the standards: ratio, relationship between quantities, for every ___ there was ____, unit rate a/b, and ratio a:b with b=0.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Ratio
- Unit Rate

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work

ELLs may need support with the following vocabulary words during the classroom discussion:

- Per
- Escalator
- Meters
- Rode
- Statement
- Traveled
- Apply
- Select
- Every

ADDITIONAL THOUGHTS

As noted in the Commentary below, this task is the first in a set of three tasks. It's interesting to view the two grade six tasks side-by-side, as this task focuses primarily on conceptual understanding of ratios and rates, while Riding at a Constant Speed focuses primarily on application of ratio and rate reasoning to solve problems. The third task in this set, Molly's Run, illuminates the heightened expectations of this domain for grade 7 (i.e., students work with ratios specified by rational numbers).

For more insight into the expectations for ratio and rate reasoning in grade six, read pages 5–7 of the progression document, *6–7, Ratios and Proportional Relationships*, available at www.achievethecore.org/progressions.

For more analysis on this task from an assessment perspective, please read the Cognitive Complexity section on the Illustrative Mathematics site.

6.RP The Escalator, Assessment Variation

Task

Ty took the escalator to the second floor. The escalator is 12 meters long, and he rode the escalator for 30 seconds. Which statements are true? Select all that apply.

a. He traveled 2 meters every 5 seconds.

b. Every 10 seconds he traveled 4 meters.

c. He traveled 2.5 meters per second.

d. He traveled 0.4 meters per second.

e. Every 25 seconds, he traveled 7 meters.

Solutions

Solution: 1

This is a one-point item.

(a), (b) and (d) are all correct.

Commentary

This task is part of a joint project between Student Achievement Partners and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

This task is part of a set of three assessment tasks that address various aspects of 6.RP domain and help distinguish between 6th and 7th grade expectations.

While simply constructed, 6.RP The Escalator addresses aspects of both 6.RP.1 "Understand the concept of a ratio" and 6.RP.2 "Understand the concept of a unit rate a/b associated with a ratio a:b with b≠0, and use rate language in the context of a ratio relationship." The simple extension of a traditional multiple choice item to a "choose all that apply" allows us to ask questions about the same context from the different perspectives afforded by the different RP standards in 6th grade.

6.RP Riding at a Constant Speed addresses aspects of 6.RP.2 "Understand the concept of a unit rate a/b associated with a ratio a:b" and 6.RP.3 "Use ratio and rate reasoning to solve real-world and mathematical problems." The numbers are chosen so that it would be easy to implement this task as a fill-in-the-blank item.

On the other hand, 7.RP Molly's Run is meant to contrast directly with "6.RP Riding at a Constant Speed" as it is the natural extension of the work that students do related to 6.RP.2. In sixth grade, the standards are clear that ratios need to have whole numbers for a and b. With the introduction of rational number arithmetic in 7.NS, the standards place an emphasis on ratios that have fractions within a given ratio; 7.RP.1 requires students to "compute unit rates associated with ratios of fractions."

Cognitive Complexity

Mathematical Content

The mathematics in "6.RP The Escalator" is more complex than it appears. The distractors are placed in a particular order. Students might choose (c) after (correctly) choosing (a) because they look similar. The three correct answers are purposefully interrupted by an incorrect choice, and (e) is included for students who subtract rather than divide.

"6.RP Riding at a Constant Speed" requires students to attend to both ratios (20:150) and (150:20) and both associated unit rates $\frac{20}{150}$ and $\frac{150}{20}$ that are implicit in the given context. Thus, this task is complex for 6th grade.

"7.RP Molly's Run" is a straight-forward extension of the work that students do in 6th grade. The only difference is that students now work with ratios defined by fractions rather than just whole numbers. Thus, this task is not mathematically complex except for students who are still struggling with fractions.

Mathematical Practice

Especially in 6th grade, the cognitive load associated with making sense of units in proportional relationships is heavy; the first two tasks in this set engage MP6, Attend to precision.

The third task does not engage any of the MPs any more than they are present in the day-to-day mathematical work of students.

Linguistic Demand

The linguistic demand for all three tasks is low.

Stimulus Material

The stimulus material for all three tasks is not complex.

Response Mode

The response mode for all three tasks is not complex.

6.RP Ticket Booth

Alignments to Content Standards: 6.RP.A.2 6.RP.A.3.a

Task

A school carnival ticket booth posts the following sign:

a. Which amount of tickets offers the best deal? Explain.

b. How would you suggest the students running the ticket booth modify the list of prices?

IM Commentary

The goal of this task is to compare unit rates in a real world context. Some of the numbers, for example 12 tickets for $5, do not give a whole number of cents per ticket. There are, however, good methods to compare this rate to the other rates without actually calculating the unit rate. In addition to solving the problem by finding unit rates, students could also make a ratio table. One advantage to the ratio table is that it supports the possibility of finding a unit rate while also allowing other thinking. For small numbers of tickets, students could use a tape diagram (or a double number line) but the size of the different groups of tickets make this challenging to do accurately.

This task was based upon an image shown here as taken from Robert Kaplinsky's blog http://robertkaplinsky.com/carnival-ride which contains student work and many other interesting insights about this problem:

This photograph was not used for the task statement because of the inappropriate use of the equals sign: the tickets *cost* a certain amount of money rather than *equal* that amount of money. In general, an equals sign should only be used with other mathematical symbols. Alternatively, teachers could use this picture and have students critically analyze how the sign has been laid out as well as solve the problem. This task has been developed in collaboration with a group of teachers from Washington and Illinois. This task was written as part of a collaborative project between Illustrative Mathematics, the Smarter Balanced Digital Library, the Teaching Channel, and Desmos.

Solutions

Edit this solution

Solution: 6.RP.A.2 Finding the price per ticket

a. For some of the groups of tickets, the price per ticket is a whole number of cents. With 25 tickets for \$10, the tickets will cost $\$10 \div 25$ each. Since \$10 is the same as 1000 cents each ticket costs 1000 ÷ 25 = 40 cents. Similarly with \$25 for 50 tickets this is the same as 2500 cents for 50 tickets or 2500 ÷ 50 = 50 cents per ticket. So the 50 tickets cost the same per ticket as the individual tickets while the groups of 25 tickets cost less per ticket. The other two possibilities, 12 tickets for \$5 and 120 tickets for \$50 are the same price per ticket because there are 10 groups of 12 tickets in 120 tickets and 10 groups of \$5 in \$50. The price per ticket is not a whole number of cents because \$5 is 500 cents and 500 is not evenly divisible by 12. The quotient is $41\ \frac{2}{3}$ cents so the price per ticket when we buy 5 or 50 tickets is a little less than 42 cents per ticket. The cheapest tickets are the groups of 25, followed next by the groups of 12 and 120, with the most expensive tickets being the individual ones and the groups of 50.

Here is all of the information from the previous paragraph in a table:

Group size of tickets	Total price (in dollars)	Price per ticket (in cents)
1	0.5	50
12	5	$41\frac{2}{3}$
25	10	40
50	25	50
120	50	$41\frac{2}{3}$

b. It is in the interest of the carnival hosts to sell as many tickets as possible. One way to encourage this is to make the larger groups of tickets less expensive per ticket. The way they have set it up, the groups of 25 tickets offer the best value. No one should purchase groups of 50 tickets because it is a better deal to buy two groups of 25 tickets and similarly no one should purchase groups of 120 tickets because it is a better deal to purchase 5 groups of 25 tickets.

Edit this solution

Solution: 6.RP.A.3.a Ratio Tables

a. One way to compare the ticket prices is with ratio tables. For example, to compare 1 ticket for 50 cents with 12 tickets for $5, we have

Number of Tickets Purchased	50 cents per ticket	$5 for 12 tickets
1	$0.50	--
12	$6	$5

Since it costs more to buy 12 individual tickets than to buy 12 tickets for $5, the tickets priced at $5 for 12 are a better deal. Notice that we could also calculate the blank spot in the table for the price of a single ticket when we buy a batch of 12 tickets for $5. This is $5 ÷ 12 = $41\frac{2}{3}$ cents, less than the 50 cents for buying a single ticket. So again, the tickets are cheaper if we buy them in groups of 12 than buying them individually.

To compare the tickets priced at $5 for 12 with those priced at $10 for 25 we have:

Number of Tickets Purchased	$5 for 12 tickets	$10 for 25 tickets
12	$5	--
24	$10	--
25	--	$10

Since we get one extra ticket for $10 when we by a group of 25 tickets, this is a better deal than buying 2 groups of 12 tickets for $5. Alternatively we could continue the table, finding a common multiple of 12 and 25:

Number of Tickets Purchased	$5 for 12 tickets	$10 for 25 tickets
300	25 × $5 = $125	12 × $10 = $120

Again, the tickets that cost \$10 for 25 are a better deal than the tickets at \$5 for 12. There are many other options available here to compare these two ways of buying tickets: we could find the price per ticket in each scenario (which is done in the first solution). Or we could find the price for 12 tickets in the 25 ticket option, namely $\frac{12}{25} \times$ \$10 = \$4.80.

The other two options for buying tickets, 50 tickets for \$25 and 120 tickets for \$50 are equivalent to ratios we have already seen. Paying \$25 for 50 tickets is the same as buying 50 individual tickets for \$0.50 while buying 120 tickets for \$50 is the same as buying 10 groups of 12 tickets for \$5.

The ratio table method and the unit rate method come together if we look for the price of 1 ticket in each of the different scenarios. On the other hand, the ratio table method is more flexible as we can compare the prices for any number of tickets purchased via the different methods.

Riding at a Constant Speed, Assessment Variation

Sample task from achievethecore.org
By Illustrative Mathematics and Student Achievement Partners

GRADE LEVEL Sixth

IN THE STANDARDS 6.RP.A.2, 6.RP.A.3

WHAT WE LIKE ABOUT THIS TASK

Mathematically:

- Provides a simply stated, yet mathematically rich task.
- Requires students to think about proportional relationships flexibly and with different units.
- Engages students with precise use of language (MP6) while interpreting ratios mathematically and in the context of the task (MP2).

In the classroom:

- Offers teachers an opportunity to analyze the student work required to answer this task.
- Gives students the opportunity to thoughtfully select the method they will use to solve the task (e.g., table of equivalent ratios, plotting points in the coordinate plane, double number line diagrams, equations) (MP5).
- Allows for both individual and group work, depending on the needs of the class.

This task was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction. Go here to learn more about the research behind these supports. This lesson aligns to ELL best practice in the following ways:

- Provides opportunities for students to practice and refine their use of mathematical language.
- Allows for whole class, small group, and paired discussion for the purpose of practicing with mathematical concepts and language.
- Includes a mathematical routine that reflects best practices to supporting ELLs in accessing mathematical concepts.
- Provides students with support in negotiating written word problems through multiple reads and/or multi-modal interactions with the problem.
- Develops meta-awareness of the language used in mathematical questions and problems.

MAKING THE SHIFTS[1]

Focus	Belongs to the Major Work[2] of grade 6
Coherence	Provides foundational work for learning about proportional relationships in grade 7 (see Molly's Run)
Rigor[3]	Conceptual Understanding: primary in this task Procedural Skill and Fluency: not targeted in this task Application: primary in this task

[1]For more information read Shifts for Mathematics.
[2]For more information, see Focus in Grade Six.
[3]Tasks will often target only one aspect of rigor.

INSTRUCTIONAL ROUTINE

The steps in this routine are adapted from the Principles for the Design of Mathematics Curricula: Promoting Language and Content Development.

Engage students in the Co-Craft Questions and Problems Mathematical Language Routine as they work through this task to allow students to get inside the context before the added pressure of producing answers.

Students will develop meta-awareness of the language used in mathematical questions and problems. Teachers support this by pushing for clarity and revoicing oral responses. Students will need to attend to both ratios (20:150 and 150:20) and both associated unit rates 20/150 and 150/20 in this task.

Co-Craft Questions:

1. Present Situation: Lin rode a bike 20 miles.
2. Students Write: Students write questions that could be answered by doing math (particularly around ratio reasoning). They can also ask questions about the situation. These could include context questions, information that is missing, or important assumptions.
3. Pairs compare: In pairs, students compare their questions.
4. Students Share: Students share their questions and briefly discuss.
5. Reveal: Share that Lin rode 20 miles in 150 minutes at a constant speed. Pause to allow students to determine if that information can help them solve the questions they generated. Finally, share the four questions from the task. Students should then solve the four questions and have an opportunity to discuss their solutions strategies with partners or in a full class discussion

Optional: After solving and discussing, support students as they co-craft problems similar to this task.

Co-Craft Problems:

1. Pairs Create New Problems: With a partner, students co-create problems that are similar to this task.
2. Students Solve their own problems: Students will solve their own problems using solution strategies and representations that make sense to them.
3. Exchange Problems: Student solve the problems created by other pairs. They then share their solutions and methods with the pair who created the problem.
4. Topic Support: If necessary, the class can brainstorm possible contexts of interest before pairing up.

LANGUAGE DEVELOPMENT

Ensure students have ample opportunities in instruction to read, write, speak, listen, and understand the mathematical concepts that are represented by the following terms and concepts:

- Constant Speed
- Per

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work

ELLs may need support with the following vocabulary words during the classroom discussion:

- Far
- Long
- Fast
- Pace

ADDITIONAL THOUGHTS

As noted in the Commentary below, this task is the second in a set of three tasks. It's interesting to view the two grade six tasks side-by-side, as The Escalator focuses primarily on conceptual understanding of ratios and rates, while this task, Riding at a Constant Speed, focuses primarily on application of ratio and rate reasoning to solve problems. The third task in this set, Molly's Run, illuminates the heightened expectations of this domain for grade 7 (i.e., students work with ratios specified by rational numbers).

For more insight into the expectations for ratio and rate reasoning in grade six, read pages 5–7 of the progression document, *6–7, Ratios and Proportional Relationships*, available at www.achievethecore.org/progressions.

For more analysis on this task from an assessment perspective, please read the Cognitive Complexity section on the Illustrative Mathematics site.

6.RP Riding at a Constant Speed, Assessment Variation

Task

Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed,

a. How far did she ride in 15 minutes?

b. How long did it take her to ride 6 miles?

c. How fast did she ride in miles per hour?

d. What was her pace in minutes per mile?

Commentary

This task is part of a joint project between Student Achievement Partners and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

This task is part of a set of three assessment tasks that address various aspects of 6.RP domain and help distinguish between 6th and 7th grade expectations.

While simply constructed, 6.RP The Escalator addresses aspects of both 6.RP.1 "Understand the concept of a ratio" and 6.RP.2 "Understand the concept of a unit rate a/b associated with a ratio a:b with b≠0, and use rate language in the context of a ratio relationship." The simple extension of a traditional multiple choice item to a "choose all that apply" allows us to ask questions about the same context from the different perspectives afforded by the different RP standards in 6th grade.

6.RP Riding at a Constant Speed addresses aspects of 6.RP.2 "Understand the concept of a unit rate a/b associated with a ratio a:b" and 6.RP.3 "Use ratio and rate reasoning to solve real-world and mathematical problems." The numbers are chosen so that it would be easy to implement this task as a fill-in-the-blank item.

On the other hand, 7.RP Molly's Run is meant to contrast directly with "6.RP Riding at a Constant Speed" as it is the natural extension of the work that students do related to 6.RP.2. In sixth grade, the standards are clear that ratios need to have whole numbers for a and b. With the introduction of rational number arithmetic in 7.NS, the standards place an emphasis on ratios that have fractions within a given ratio; 7.RP.1 requires students to "compute unit rates associated with ratios of fractions."

Cognitive Complexity

Mathematical Content

The mathematics in "6.RP The Escalator" is more complex than it appears. The distractors are placed in a particular order. Students might choose (c) after (correctly) choosing (a) because they look similar. The three correct answers are purposefully interrupted by an incorrect choice, and (e) is included for students who subtract rather than divide.

"6.RP Riding at a Constant Speed" requires students to attend to both ratios (20:150) and (150:20) and both associated unit rates $\frac{20}{150}$ and $\frac{150}{20}$ that are implicit in the given context. Thus, this task is complex for 6th grade.

"7.RP Molly's Run" is a straight-forward extension of the work that students do in 6th grade. The only difference is that students now work with ratios defined by fractions rather than just whole numbers. Thus, this task is not mathematically complex except for students who are still struggling with fractions.

Mathematical Practice

Especially in 6th grade, the cognitive load associated with making sense of units in proportional relationships is heavy; the first two tasks in this set engage MP6, Attend to precision.

The third task does not engage any of the MPs any more than they are present in the day-to-day mathematical work of students.

Linguistic Demand

The linguistic demand for all three tasks is low.

Stimulus Material

The stimulus material for all three tasks is not complex.

Response Mode

The response mode for all three tasks is not complex.

Solutions

Solution: 1

	A	B	C	D	E	F
Number of Minutes	150	15	7.5	30	45	60
Number of Miles	20	2	1	4	6	8

The values in column B were found by dividing both values in column A by 10. The values in column C were found by dividing both values in column B by 2. The other columns contain multiples of the values in column B.

a. If we look in column B, we can see that she could ride 2 miles in 15 minutes.

b. If we look in column E, we can see that it would take her 45 minutes to ride 6 miles.

c. If we look in column F, we can see that she is riding 8 miles every 60 minutes (which is 1 hour), so she is riding her bike at a rate of 8 miles per hour.

d. If we look in column C, we can see that her pace is 7.5 minutes per mile.

This is a four-point item.

Solution: 2

a. She could ride 1 mile in 7.5 minutes and 2 miles (1 + 1) in 15 minutes (7.5 + 7.5).

b. She rides 150/20 minutes per mile which is 7.5 minutes per mile. So it would take her 45 minutes to ride 6 miles because 6 × 7.5 = 45.

c. If she rides 2 miles in 15 minutes, then she can ride 4 miles in 30 minutes and 8 miles per hour. d. She rides 7.5 minutes per mile.