Grade 7
Family Resource Bundle

Grade 7

ANSWER KEY Text #1 Door to Freedom

by Jacalyn McNamara 1982

1. RI.KID.2

PART A: Which statement describes the main idea of the memoir?

A. Pal refused to give up on going to America, even when it looked like he would never get his immigration papers.
B. In order to discourage people from immigrating to America, the embassy made it difficult for people to get their immigration papers.
C. America is the best place for young people to start over if they have lost everything because of war.
D. It's not fair that Pal was able to get his immigration papers by breaking the rules, while most people had to wait in line.

2. RI.KID.1

PART B: Which detail from the text best supports the answer to Part A?

A. "'If I were a young man, I'd go to America. It's the land of the future. There, a man can be free. You should go to America.'" (Paragraph 7)
B. "By six o'clock in the evening, Pal was still far back in the line when two marines closed the big doors. Cold and in despair, Pal returned to the home of his grandfather's friend." (Paragraph 10)
C. "Tea burned his lips but aroused his courage. There had to be another way into the embassy, he thought, and he had to find it." (Paragraph 16)
D. "As he lay on the straw he dreamed of thousands of people frozen like icicles in the street." (Paragraph 26)

3. RI.KID.3

PART A: How does the author's account of refugees' experiences help us understand the journey to America as a refugee?

A. It emphasizes how difficult it was to enter the embassy and obtain papers to immigrate to America as a refugee.
B. It suggests that refugees needed a lot of money to afford to leave their country and make a new life in America.
C. It shows how embassy officials made it difficult for refugees to come to America on purpose because they didn't like immigrants.
D. It suggests that you needed special connections with people in the embassy to get to America.

4. RI.KID.1

PART B: Which quote from the text best supports the answer to Part A?

A. **"At the embassy the lines had grown. Would the quota be filled before he could even get in? He wondered." (Paragraph 12)**
B. "Marines guarded the front doors all day. East of the building was a delivery area surrounded by a ten-foot railing." (Paragraph 17)
C. "'Who is sponsoring you?' the secretary asked. He remembered that his grandfather had donated to the organization." (Paragraph 22)
D. "He led the boy to the stairs. 'Just walk through and say hello. Don't tell anyone else about it until you have your papers or they might lock the door.'" (Paragraph 29)

5. RI.CS.5

How do paragraphs 28-29 contribute to the development of ideas in the text?

1. **Answers will vary; students should discuss how the final two paragraphs of the text demonstrate Pal's desire to help others have the same good fortune he had. When Pal got his immigration papers, the author described him as feeling "hollow" after seeing the other refugees waiting in line (Paragraph 28). In order to help other refugees get their immigration papers, Pal told a boy, "'Just walk through and say hello. Don't tell anyone else about it until you have your papers or they might lock the door'" (Paragraph 29). Pal realized how difficult it can be to get immigration papers, and saw it as his duty to help other refugees accomplish what he did. In the end, Pal gave the boy his money and told him "'Maybe we'll be neighbors in America'" (Paragraph 29). In all, the last two paragraphs reiterate how difficult it can be for refugees to get their immigration papers and show Pal's desire to help others who shared his goal: getting to America.**

ANSWER KEY Text #2 Mother to Son

by Langston Hughes 1922

1. RL.KID.3

In the poem, whom is the speaker addressing and about what?

A. A mother is telling a story to her child about her own childhood.
B. A mother is describing for her son the climb up a crystal staircase.
C. A son is recounting a conversation with his mother about his struggle to earn a comfortable living.
D. **A mother is warning her son about the difficulties of life and the struggle to persevere.**

2. RL.CS.4

Which of the following best explains the significance of the staircase in the poem?

A. **The narrator describes a tiring climb up a beat-up staircase, which represents her persistence through difficulties and challenges in life.**
B. The narrator describes a crystal staircase, which symbolizes her goals and the hard work she has done to accomplish her dreams.

C. The narrator describes her climb up a dirty staircase that transforms into a crystal stair, which represents her ability to rise above difficulties.
D. The narrator describes herself going down a staircase that is falling apart, which represents her fleeing a difficult life.

3. RL.KID.2

PART A: Which of the following statements best describes a major theme of the poem?

A. Never forget your family.
B. Persevere when life isn't easy.
C. Hope is the answer to all challenges.
D. Respect your elders.

4. RL.KID.1

PART B: Which of the following quotes best supports the answer to Part A?

A. "Well, son, I'll tell you: / Life for me ain't been no crystal stair." (Lines 1-2)
B. "It's had tacks in it, / And splinters, / And boards torn up, / And places with no carpet on the floor — / Bare." (Lines 3-7)
C. "I'se been a-climbin' on, / And reachin' landin's, / And turnin' corners" (Lines 9-11)
D. "So boy, don't you turn back. / Don't you set down on the steps / 'Cause you finds it's kinder hard. / Don't you fall now — / For I'se still goin', honey" (Lines 14-18)

5. RL.CS.5

RL.CS.6

How does the poem's use of language and free verse contribute to the author's purpose?

1. **Answers will vary; students should discuss how the free verse style and Hughes' use of the vernacular, or the mimicking of everyday speech patterns and conversation, contribute to the author's purpose to accurately portray a heartfelt conversation between a mother and son. By reflecting natural speech patterns in his poem, the author creates a sense of earnestness and sincerity that makes the speaker's message to keep climbing even more poignant.**

Related Media Links and Descriptions

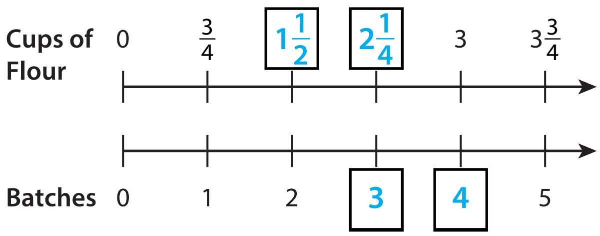
Related Media #1: What Does it Mean to Be a Refugee?- Benedetta Berti and Evelien Borgman
Show this video to students to provide them with information about refugees. 5:42

Related Media #2: Langston Hughes- Biography
Langston Hughes is well-known for bringing African American culture and identity to a wider audience during the Harlem Renaissance. In this biographical video, historians and experts explain how Hughes was "one of the early figures to show the dignity and beauty of ordinary black life." 3:33

Understanding Proportional Relationships

➤ **Read and solve the problems. Show your work.**

1. Josie is making pizza dough. Complete the double number line by filling in the missing values. Then write an equation that models the relationship between the total cups of flour, *c*, and number of batches, *n*. Show your work.



$c = \frac{3}{4}n$

2. Lilli bought each of her friends a pair of colorful socks that cost $5.50. Complete the table to show how much Lilli paid to buy different numbers of socks. Then write an equation that shows the total cost, *c*, for *p* pairs of socks.

Cost	$5.50	$11.00	$16.50	$22.00	$27.50
Pairs of socks	1	2	3	4	5

$c = 5.5p$

3. Explain how using a table is similar to using a double number line and how it is different.

Possible answer:

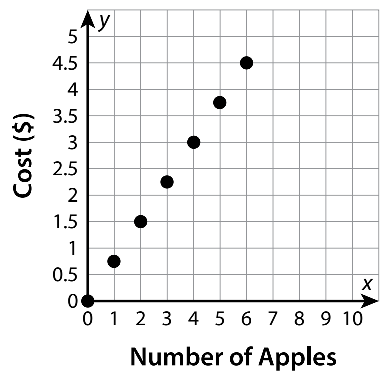
Double number lines and tables both show corresponding values in a proportional relationship. The ratios formed by corresponding values are always equivalent in both a table and a double number line. A double number line usually starts at 0 and increases incrementally. A table does not necessarily start at 0 and may not increase incrementally.

4. Mrs. Lopez types at a constant rate. The constant of proportionality for the relationship between the number of words she types, *w*, and the number of minutes she types, *m*, is 38. Write an equation to show this relationship.

$w = 38m$

Interpreting Graphs of Proportional Relationships

➤ **The graph shows the cost of apples at a local market. Use the graph to answer problems 1–3.**



1. What is the cost of 1 apple and of 3 apples? How do you know?

   Possible answer: One apple costs \$0.75, and 3 apples cost \$2.25. The points (1, 0.75) and (3, 2.25) are on the graph. The *x*-coordinate of 1 corresponds to the *y*-coordinate of 0.75, and the *x*-coordinate of 3 corresponds to the *y*-coordinate of 2.25.

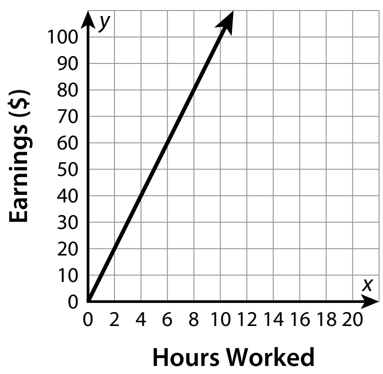
2. What does the point (0, 0) represent in this context?

   Possible answer: (0, 0) means that 0 apples cost \$0.00.

3. What does the point (2, 1.5) represent in this context?

   Possible answer: The cost of 2 apples is \$1.50.

➤ **The graph shows Manuela's earnings for the number of hours she spends tutoring. Use the graph to answer problems 4 and 5.**



4. How much does Manuela earn for each hour of tutoring? Explain.

   Possible answer:

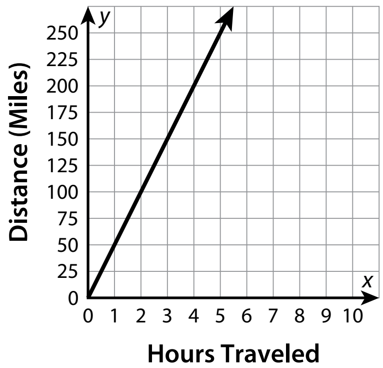
   \$10 per hour; Possible explanation: The graph goes through the point (1, 10). The *y*-coordinate associated with the *x*-coordinate of 1 is the constant of proportionality.

5. Write an equation that shows the relationship between Manuela's earnings, *y*, and hours, *x*.

   $y = 10x$

Interpreting Graphs of Proportional Relationships *continued*

➤ **The graph shows the distance Jason's family traveled on a recent road trip. Use the graph to answer problems 6–8.**



6 What is the constant of proportionality? Explain how you know.

50; Possible explanation: The point (1, 50) is on the graph. The *y*-coordinate associated with the *x*-coordinate of 1 is the constant of proportionality.

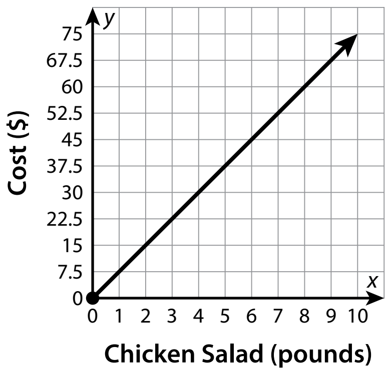
7 Identify and interpret one other point on the graph.

Possible answer: The point (2, 100) means that Jason's family traveled 100 miles in 2 hours.

8 Write an equation that models the distance, *d*, traveled in *t* hours.

$d = 50t$

➤ **The graph shows the cost per pound of chicken salad. Use the graph to answer problems 9 and 10.**



9 Randy claims that he can purchase 3.5 pounds of chicken salad for $23.50. Is he correct? Explain.

No; Possible explanation: According to the graph, 3.5 corresponds to the point halfway between 22.5 and 30, and 23.5 is not halfway.

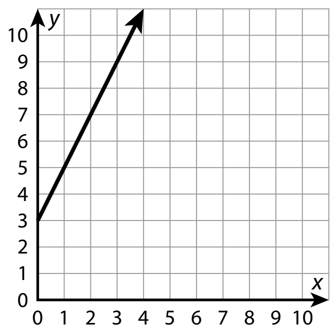
10 Explain how you can determine how much chicken salad may be purchased for $52.50.

Possible answer: You can find the *x*-coordinate that corresponds with the *y*-value of 52.5 on the graph.

Recognizing Graphs of Proportional Relationships

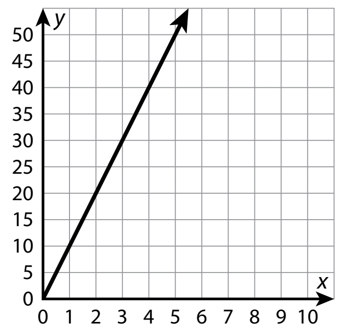
➤ **Circle all the problems with graphs that do NOT represent a proportional relationship. For the problems that are circled, explain why the graphs do not represent a proportional relationship.**

1



The graph does not go through the origin.

2



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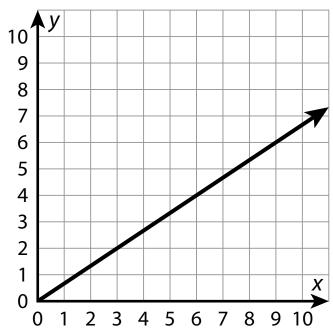

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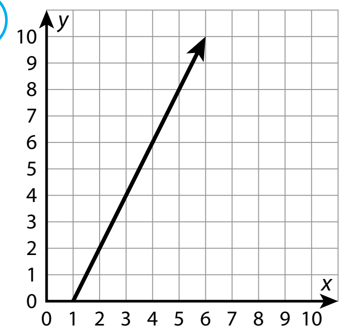
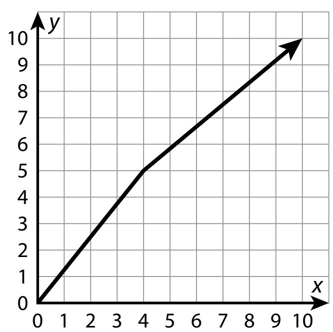


The graph does not go through the origin.

5



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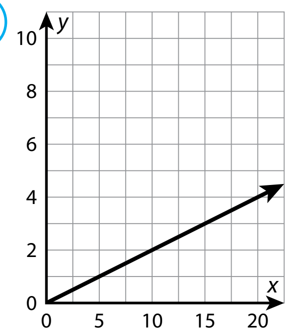
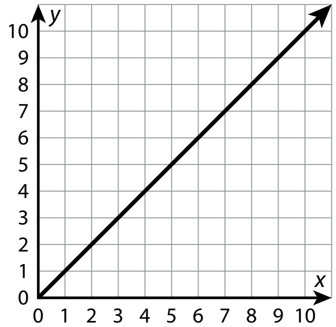


The *x*-values do not change as the *y*-values increase.

Recognizing Graphs of Proportional Relationships *continued*

7



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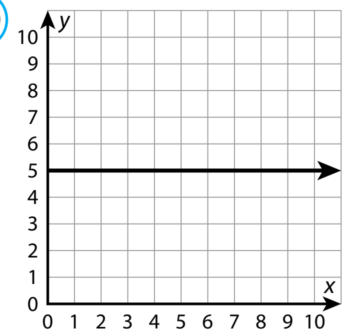
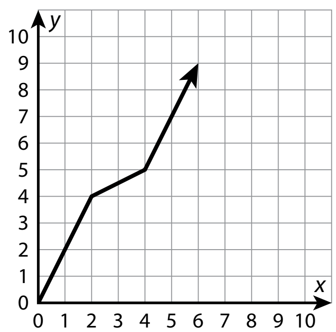


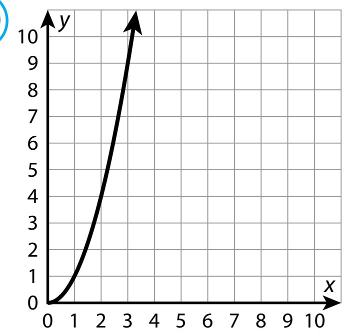
The *y*-values do not change as the *x*-values increase.

9



The graph is not a straight line.

10



The graph is not a straight line.

11 Without analyzing specific points on a graph, explain how you know whether a graph shows a proportional relationship.

Possible answer: The graph of a proportional relationship is a straight line that passes through the origin, with all points on the line representing equivalent ratios.

Solving Multi-Step Ratio Problems

➤ **Solve each problem.**

1. At The Green House of Salad, you get a $1 coupon for every 3 salads you buy. What is the least number of salads you could buy to get $10 in coupons?

   30 salads

2. Kim orders catering from Midtown Diner for $35. She spends $5 on a large order of potato salad and the rest on turkey sandwiches. Each sandwich is $2.50. How many sandwiches does Kim buy?

   12 sandwiches

3. Molly and Liza are exercising. Molly does 10 push-ups at the same time as Liza does 15 push-ups. When Molly does 40 push-ups, how many push-ups does Liza do?

   60 push-ups

4. A shark swims at a speed of 25 miles per hour. The shark rests after 40 miles. How long, in minutes, does the shark swim before resting?

   96 minutes

5. Ali and Janet are selling gifts at a local craft show. For every bar of soap that Ali sells, she earns $5. For every mug that Janet sells, she earns twice as much as Ali. Ali sells 5 bars of soap, and Janet sells 7 mugs. How much money did they make altogether?

   $95

6. Ted is making trail mix for a party. He mixes $1\frac{1}{2}$ cups of nuts, $\frac{1}{4}$ cup of raisins, and $\frac{1}{4}$ cup of pretzels. How many cups of pretzels does Ted need to make 15 cups of trail mix?

   $1\frac{7}{8}$ cups of pretzels

7. The ratio of chaperones to students on a field trip is 2 : 7. There are 14 chaperones on the field trip. In all, how many chaperones and students are there?

   63 students and chaperones

8. Dayren is driving to visit family. She drives at an average of 65 miles per hour. She drives 227.5 miles before lunch and then 97.5 miles after lunch. How many hours did she spend driving?

   5 hours

Solving Problems Involving Multiple Percents

➤ **Solve each problem.**

1. A chair's regular price is $349. It is on clearance for 30% off, and a customer uses a 15% off coupon after that. What is the final cost of the chair before sales tax?

   $207.66

2. A calculator is listed for $110 and is on clearance for 35% off. Sales tax is 7%. What is the cost of the calculator?

   $76.51

3. Cara started working for $9 per hour. She earns a 4% raise every year. What is her hourly wage after three years?

   $10.12 per hour

4. A factory manufactures a metal piece in 32 minutes. New technology allowed the factory to cut that time by 8%. Then another improvement cut the time by 5%. How long does it take to manufacture the piece now? Round your answer to the nearest minute.

   28 minutes

5. An apartment costs $875 per month to rent. The owner raises the price by 20% and then gives a discount of 8% to renters who sign an 18-month lease. How much less do renters who sign an 18-month lease pay per month to rent the apartment?

   $84 less

Solving Problems Involving Multiple Percents *continued*

6 Damon buys lumber worth $562. He gets a 20% contractor's discount. The sales tax is 6%. His credit card gives him 2% off. How much does he pay?

$467.04

7 Cindy is shopping for a television. The original price is $612. Store A has the television on clearance for 30% off. Store B has it on clearance for 25% off, and Cindy has a 10% off coupon to use at Store B. At which store will she pay less? How much less?

Store B; $15.30 less

8 John goes to a restaurant and has a bill of $32.57. He uses a 10% off coupon on the cost of the meal. The tax is 8%. He leaves a tip of 18% on the amount before the coupon or tax is applied. How much does he spend?

$37.52

9 Explain which situation will give you the best price: a discount of 15% and then 10% off that amount, a discount of 10% and then 15% off that amount, or a discount of 25%.

a discount of 25%; Possible explanation: Applying a 15% off discount and a 10% off discount in either order results in the same final amount because of the commutative property of multiplication. This final amount is more than when a 25% off discount is applied.

Solving Problems Involving Percent Change

➤ **Find the percent change and tell whether it is a percent increase or a percent decrease.**

1. Original amount: 20
End amount: 15

25% decrease

2. Original amount: 30
End amount: 45

50% increase

3. Original amount: 625
End amount: 550

12% decrease

4. Original amount: 320
End amount: 112

65% decrease

5. Original amount: 165
End amount: 222.75

35% increase

6. Original amount: 326
End amount: 423.80

30% increase

7. Original amount: 27
End amount: 38.61

43% increase

8. Original amount: 60
End amount: 70.02

16.7% increase

9. How do you know when a situation involves a percent increase or a percent decrease?

Possible answer: When the end amount is greater than the original amount, there is a percent increase. When the end amount is less than the original amount, there is a percent decrease.

Solving Problems Involving Percent Error

➤ **Solve each problem. Round to the nearest hundredth of a percent if needed.**

1. Mrs. Rowan allotted 30 minutes at the beginning of class for her students to complete an exam. The last student took 42 minutes to complete the exam. What is Mrs. Rowan's percent error?

   40%

2. Harper needs to mail an envelope. She weighs it at home as 10.4 ounces. When she gets to the post office, the clerk weighs it at 9.6 ounces. What is the percent error in the weight of the envelope?

   7.69%

3. An airline ticket states that the flight takes 2 hours and 45 minutes. The flight time is actually 2 hours and 54 minutes. What is the percent error in the flight time?

   5.45%

4. Luna buys a shirt that costs $15.65. She gives the cashier $20 and receives $3.25 in change. What is the percent error in the amount of change she was given?

   25.29%

5. Judy thinks there will be 325 people at the county fair on Friday, while Atticus thinks there will be 600 people. On Friday, 452 people attend the fair. Who is closer in their estimate? What is the difference between the percent errors?

   Judy is closer by about 4.64%.

6. Sussex County received 43 inches of rainfall this year. The percent error in the local meteorologist's rainfall prediction was about 18.02%. What are two possible values for the meteorologist's prediction?

   35.25 inches, 50.75 inches

7.RP Perfect Purple Paint II

Alignments to Content Standards: 7.RP.A

Task

Jessica gets her favorite shade of purple paint by mixing 1/3 cup of blue paint with 1/2 cup of red paint. How many cups of blue and red paint does Jessica need to make 20 cups of her favorite purple paint?

IM Commentary

The goal of this task is to provide a context for students to develop their ratio and proportional reasoning skills. Many of the techniques developed in the sixth grade can be readily applied here: for example, solution 2 below uses a ratio table. Other techniques, such as double number lines or tape diagrams, can also be applied (see solution 6) although the fractions make these more challenging. If the fractions in the problem (1/3 and 1/2) were more complex (1/3 and 1/5 for example) then there is genuine motivation to examine more abstract techniques. The numbers chosen make this a good bridge problem, allowing students to practice the techniques learned in the sixth grade while also working with the more abstract seventh grade ideas.

The sixth grade version of this problem, https://www.illustrativemathematics.org/tasks/2049, replaces the fractions 1/3 and 1/2 in the prompt with whole numbers 2 and 3. If students are successful applying double number lines, tape diagrams, and ratio tables with fractions then they have demonstrated mastery of the sixth grade techniques and are ready to move into the more abstract ratio and proportion methods exemplified by solutions 3 and 4.

If this task is being used to motivate proportional reasoning techniques, the teacher may wish to give students more complex fractions than 1/2 and 1/3. As the numbers become more complex, physical representations such as tape diagrams and double number lines become more difficult to manipulate. Methods such as scaling (solution 1), setting up a proportion (solution 3), or setting up an equation (solution 4) work for any fractions, provided students are fluent doing fraction arithmetic.

In addition to being used to bridge sixth and seventh grade ratio and proportional thinking, this task can also be used after students have experience with the seventh grade language and techniques. In this case, a large variety of responses, as shown, should be encouraged and expected and the teacher may wish to have students share their different approaches. The vast array of techniques available to solve this problem are indicative of the central role of ratio and proportion in the middle school curriculum. Building upon arithmetic with fractions, it prepares students for using expressions and equations, graphing lines, and understanding the meaning of functions.

This task was developed with the assistance of a group of teachers from Washington and Illinois in connection with an SBAC digital library project. In the lesson, the statement of the problem was ''If Perfect Purple Paint is made by mixing 1/3 cups blue paint to 1/2 cup red paint, how much of each is needed for 20 cups?''

This task was written as part of a collaborative project between Illustrative Mathematics, the Smarter Balanced Digital Library, the Teaching Channel, and Desmos.

Solutions

Edit this solution

Solution: 1 Arithmetic and Scaling

One batch of purple paint contains 1/3 cup of blue paint and 1/2 cup of red paint. This will make a total of 1/2 + 1/3 = 5/6 of a cup of purple paint. In order to make 20 cups of purple paint, we need 20 ÷ 5/6 = 24 batches. Each batch has 1/3 of a cup of blue paint so 24 batches will contain 24 × 1/3 = 8 cups of blue paint. Each batch has 1/2 cup of red paint so 24 batches will contain 24 × 1/2 = 12 cups of red paint.

Edit this solution

Solution: 2 Ratio table

We can use a ratio table to find how much blue and red paint will be in 20 cups of

Jessica's perfect purple paint:

Blue Paint (cups)	Red Paint (cups)	Purple Paint (cups)
1/3	1/2	5/6
2	3	5
4	6	10
8	12	20

For the second row, we take 6 batches of the purple paint mixture, in order to get a whole number of cups of purple paint. From here, doubling this mixture twice shows that there are 8 cups of blue paint and 12 cups of red paint in 20 cups of purple paint.

Edit this solution

Solution: 3 Proportions

Combining 1/3 cup of blue paint and 1/2 cup of red paint makes 1/3 + 1/2 = 5/6 cups of blue paint. To find how much blue paint is in 20 cups of purple paint we can use a proportion:

$$\frac{1}{3} : \frac{5}{6} :: ? : 20.$$

Note that $\frac{1}{3} = \frac{2}{6}$ so $\frac{1}{3} \div \frac{5}{6} = \frac{2}{6} \div \frac{5}{6} = \frac{2}{5}$. So ? satisfies the equation

$$\frac{?}{20} = \frac{2}{5}.$$

We can solve this equation to find ? = 8. There are 8 cups of blue paint in 20 cups of Jessica's purple paint.

Since the rest of the paint in the 20 cups of purple paint is red, this means that there are 12 cups of red paint in the purple paint mixture.

Edit this solution

Solution: 4 Equations

One batch of purple paint is 1/3 + 1/2 = 5/6 cups. The amount of blue paint is 1/3 = 2/6 cups. This means that there is 5/2 times as much purple paint as blue paint. If p is the amount of purple paint and b the amount of blue paint, this means

$$p = \frac{5}{2}b.$$

So if we have 20 cups of purple paint then to find out how much blue paint there is we can solve

$$20 = \frac{5}{2}b$$

to find that there are 8 cups of blue paint.

Similarly, if r denotes the red paint in the mixture then

$$p = \frac{5}{3}r$$

and if $p = 20$ then we find $r = 12$ so there are 12 cups of red paint in 20 cups of Jessica's purple paint.

Edit this solution

Solution: 5 Using Percent

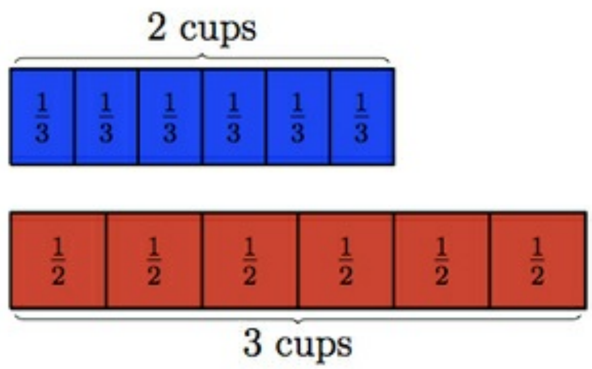
There are 1/3 + 1/2 = 5/6 cups of purple paint in a single batch of Jessica's favorite purple. Since 1/3 = 2/6 this means that 2 out of 5 equal parts, or 40%, of the purple paint comes from blue paint. Since 40% of 20 cups is 8 cups, 8 cups of blue paint are needed to make 20 cups of purple paint.

Similarly, the remaining 60% of the purple paint comes from the added red paint. Since 60% of 20 cups is 12 cups, 12 cups of red paint are needed to make 20 cups of purple paint.

Edit this solution

Solution: 6 Tape diagrams

We put together batches of the purple paint until we find whole number of cups of both red paint and blue paint:



Here we have taken 6 batches, each consisting of 1/3 cup of blue paint and 1/2 cup of red paint. Together, this makes 5 cups of purple paint. So if we combine four of these that makes 20 cups of perfect purple paint. Four groups of 2 cups of blue paint make 8 cups of blue paint and four groups of 3 cups of red paint make 12 cups. So 20 cups of perfect purple paint contains 8 cups of blue paint and 12 cups of red paint.

7.RP Sale!

Alignments to Content Standards: 7.RP.A

Task

Four different stores are having a sale. The signs below show the discounts available at each of the four stores.

Two for the price of one	Buy one and get 25% off the second
Buy two and get 50% off the second one	Three for the price of two

a. Which of these four different offers gives the biggest price reduction? Explain your reasoning clearly.

b. Which of these four different offers gives the smallest price reduction? Explain your reasoning clearly.

IM Commentary

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students.

The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, the commentary will spotlight one practice connection in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

The purpose of this task is to engage students in Standard for Mathematical Practice 4, "Model with mathematics," and as such, the question as it is worded cannot be answered without making some assumptions. For example, if the items that are purchased do not have the same value, then the price reduction depends on the cost of the items. The answer also depends on how you interpret the meaning of “price reduction” which could be either the absolute reduction or the relative reduction. Consider the four scenarios for purchasing pairs of shoes below.

“Two for the price of one”

Pair 1	Pair 2	Money saved	Fraction of purchase saved
\$36	\$12	\$12	$\frac{1}{4}$
\$36	\$36	\$36	$\frac{1}{2}$

“Three for the price of two”

Pair 1	Pair 2	Pair 2	Money saved	Fraction of purchase saved
\$60	\$48	\$18	\$18	$\frac{1}{7}$
\$12	\$12	\$12	\$12	$\frac{1}{3}$

Which has the greatest price reduction? It depends, and a complete answer to this question requires a mathematical argument beyond the expectations of 7th grade. On the other hand, students need opportunities to evaluate the relative savings of advertised sales, so realizing that the best sale depends on what you are buying is a good insight to develop. The solutions below assume that you are comparing the sales for purchasing items of the same price.

It is also worth pointing out that there is a very important, although non-mathematical, issue related to whether a particular sale will save you money: you do not save money by buying things you do not need. So, for example, 3 for the price of 2 is not a better deal than buy one get the second at 25% off if you do not need three of the item.l.

The teacher might use this task after formally teaching 7.RP.1-3. Students could be given the task and asked to collaborate in small groups to solve the questions posed using all the formal instruction on ratio and proportional reasoning. The teacher might ask questions such as; “What if the price of each item is different? Does that change which discount is biggest?” “What if the price of each item is the same? Does that make a difference in which discount is biggest?” Depending on the level of the students, the teacher could direct the students by giving them a specific value or use a more abstract approach by having them solve using a variable. The students could share out their findings and compare/contrast the answers and discuss why the results vary.

Solutions

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Solution: Starting with a specific value

Assume that you are comparing the sales for purchasing items of the same price (it is a much harder question to answer if you don’t). Let’s first look at the answer for a specific value. Suppose the regular price for all items is $60. Then the following table shows how much you will pay per item.

	Cost for 3 items	Cost for 2 items	Cost per item	Total savings
2 for 1		60	60 ÷ 2 = 30	$60
25% off the 2nd		60 + 45 = 105	105 ÷ 2 = 52.50	$15
50% off the 2nd		60 + 30 = 90	90 ÷ 2 = 45	$30
3 for 2	60 + 60 = 120		120 ÷ 3 = 40	$60

In general, suppose that a single item costs x dollars.

	Cost for 3 items	Cost for 2 items	Cost per item	Total savings
2 for 1		x	$x \div 2 = \frac{1}{2}x$	x

25% off the 2nd		$x+\frac{3}{4}x=\frac{7}{4}x$	$\frac{7}{4}x\div 2=\frac{7}{8}x$	$\frac{1}{4}x$
50% off the 2nd		$x+\frac{1}{2}x=\frac{3}{2}x$	$\frac{3}{2}x\div 2=\frac{3}{4}x$	$\frac{1}{2}x$
3 for 2	$x+x$		$2x\div 3=\frac{2}{3}x$	x

So “Two for the price of one” gives the biggest price reduction per item but the total savings is the same as the “Three for the price of two” sale.

Also, “Buy one and get 25% off the second” has both the highest price per item and the lowest total savings, which means it offers the smallest price reduction.

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Solution: A more abstract approach

Assume we are comparing the sales for buying identically priced items and that we are comparing the reduction in price per item (as opposed to price for the entire purchase). The sale price (per item) is the total cost divided by the number of items. The reduction in price is equal to the original price minus the sale price. We need to calculate the price reduction for every case in order to answer the two questions.

Since we don't know what the regular price per item is let's just call it p.

Two for the price of one:

The reduction is: $p-\frac{1p}{2}=\frac{1}{2}p$

Buy one and get 25% off the second:

The reduction is: $p-\frac{(1+0.75)p}{2}=p-\frac{1.75p}{2}=p-\frac{7/4}{2}p=p-\frac{7}{4}\cdot\frac{1}{2}p=p-\frac{7}{8}p=\frac{1}{8}p$

Buy two and get 50% off the second one:

The reduction is: $p-\frac{(1+0.50)p}{2}=p-\frac{1.5p}{2}=p-\frac{3/2}{2}p=p-\frac{3}{2}\cdot\frac{1}{2}p=p-\frac{3}{4}p=\frac{1}{4}p$

Three for the price of two:

The reduction is: $p - \frac{2p}{3} = \frac{1}{3}p$

Now we can answer the questions

Which of these four different offers gives the biggest price reduction?

Two for the price of one with a price reduction of one half.

Which of these four different offers gives the smallest price reduction?

Buy one and get 25% off the second with a price reduction of one eighth.

7.RP Dueling Candidates

Alignments to Content Standards: 7.RP.A

Task

Joel and Marisa are running for president at their middle school (grades 6-8). After the votes are in, Joel and Marisa are each convinced that they have won the election:

- Joel argues that he has won a larger percentage of the overall vote than Marisa so he should be the new president.
- Marisa argues that she has won a larger percentage than Joel of the 6th grade vote and the 7th grade vote. Since the majority of the grades voted for her, she should be the new president.

Is it possible that both Joel and Marisa are making accurate claims? Explain.

IM Commentary

The goal of this task is to have students examine some properties of ratios (and fractions) in an important real world context. Students will gain practice working with ratios while investigating some of the complexities of voting theory. This task can be made less open-ended by supplying, for example, the total number of students at the school or even the number of students in each class: the teacher may also wish to discuss the analysis in the first paragraph of the first solution if students are stuck. As written, it is intended to be an engaging, open-ended question and will require ample time. It is an ideal task for group work.

If an extension of the problem is desired, the teacher might ask if it is possible for Joel to win a larger percentage of the overall vote than Marisa while Marisa wins a larger percentage of the vote within *every* grade level. This scenario is impossible because it would mean that Marisa wins more votes than Joel in each grade but fewer votes when the grades are added up.

It is important for students to understand that both Joel and Marisa have legitimate arguments and so it is essential that the rules governing the election be specified in advance. For a further discussion, the teacher may wish to show students numbers for the 2000 presidential election:

http://en.wikipedia.org/wiki/United_States_presidential_election,_2000

Al Gore received *more* overall votes than George W. Bush (Joel's argument) but Bush won the election because presidential elections are determined on a state by state basis (Marisa's argument). The rules governing presidential elections follow Marisa's line of reasoning (although there are additonal weights involved as each state has a given number of delegates), where the states play the role of the different grade levels at the school. So George W. Bush was elected president even though he received fewer votes than Al Gore.

Students working on this task will engage in MP2, Reason Abstractly and Quantitatively, as the main work of the task involves constructing and reasoning with numbers which satisfy constraints (which also must be reasoned out from the context). The task also provides an opportunity to work on MP3, Construct Viable Arguments and Critique the Reasoning of Others. This may happen at two levels: first students will critique the supplied reasoning of Joel and Marisa and, secondly, they may well disagree about which line of reasoning is more convincing and then they will examine and critique the reasoning of one another.

This task was designed for an NSF supported summer program for teachers and undergraduate students held at the University of New Mexico from July 29 through August 2, 2013 (http://www.math.unm.edu/mctp/).

Solutions

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Solution: 1 Working with Fractions and Percents (6.RP.3)

We are given that Joel has won the majority of the total number of votes at the school. On the other hand, when the vote is divided up by grade level, Marisa has a higher percentage of the 6th grade and 7th grade votes. Before choosing numbers, note that if Joel wins a higher percentage of the 8th grade votes than Marisa then he has a chance to make up for the ground he lost in the 6th and 7th grade. To see if the two scenarios are consistent we have to choose numbers so that Joel makes up more ground on the 8th grade votes than he lost in the combined 6th and 7th grade votes.

We will assume for simplicity that there are 600 students at the middle school. We will also assume that all 600 students vote for either Joel or Marisa. We need to divide the 600 students between the three grades and begin with the assumption that there are 200 in each grade. As observed above, we need to make sure that Joel wins the 8th grade vote by more than Marisa's combined 6th and 7th grade wins. Suppose the votes go as in the table below:

Candidate	6th grade votes (% of 6th grade votes)	7th grade votes (% of 7th grade votes)	8th grade votes (% of 8th grade votes)
Joel	80 (40%)	90 (45%)	140 (70%)
Marisa	120 (60%)	110 (55%)	60 (30%)

We can see that Marisa won more of the 6th and 7th grade votes than Joel and so she won a larger percentage of the 6th grade votes and the 7th grade votes. For the overall vote, however, Joel received $80 + 90 + 140 = 310$ votes while Marisa received $120 + 110 + 60 = 290$ votes. Converting to percentages, we find that Joel has won

$$100 \times \left(\frac{310}{600}\right) \approx 52\%$$

of the vote while Marisa has won

$$100 \times \left(\frac{290}{600}\right) \approx 48\%$$

of the vote. So Joel has won a larger percentage of the overall vote than Marisa.

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Solution: 2 Working with ratios (7.RP.3)

We begin with a table representing the scenario described in the problem with question marks where we need to provide information:

	6th grade vote	7th grade vote	8th grade vote	total school vote

Marisa:Joel	?	?	?	?

For the 6th and 7th grade columns, we know that Marisa has won a larger percentage of those votes than Joel. For the 8th grade and total school columns, Joel has won a larger percentage of those votes than Marisa. We need to see if it is possible to assign numbers so that all 4 of these conditions hold. We begin by setting the size of each class: for simplicity, we will assume that there are 100 students in each of the 6th, 7th, and 8th grade classes and so there are 300 total students.

For the first two columns we start by putting in some numbers where the vote favors Marisa and then see if we can complete the table:

	6th grade vote	7th grade vote	8th grade vote	total school vote
Marisa:Joel	55:45	60:40	?	?

Here Marisa has won a larger percentage of the 6th grade vote (55 percent to 45 percent) and a larger percentage of the 7th grade vote (60 percent to 40 percent). Note that if Marisa had won the 6th and 7th grade votes by a margin of more than 100 votes, then it is impossible for Joel to make up the difference with the 100 votes in the 8th grade. With the values we have entered for the 6th and 7th grade, Marisa has won these two by a combined margin of 30 votes. So for Joel to win the 8th grade vote *and* the overall vote, he needs a margin of more than 30 votes in the 8th grade. Here is what we get if we assume a 70 to 30 split of the 8th grade vote in favor of Joel:

	6th grade vote	7th grade vote	8th grade vote	total school vote
Marisa:Joel	55:45	60:40	30:70	145:155

The table above gives values which match the situation described: Marisa has won a larger percentage of the 6th and 7th grade votes. The 8th grade vote splits as 70% for Joel and 30% for Marisa. There are 300 votes total so to find the percentage of the overall vote won by each candidate we can divide each number in the total vote ratio by 3:

$$\frac{145}{3} : \frac{155}{3} :: 48.\overline{3} : 51.\overline{6}.$$

Here we see that for the total vote, Marisa won a little over 48 percent and Joel won a little less than 52 percent.

7.RP Track Practice

Alignments to Content Standards: 7.RP.A 7.RP.A.1

Task

Angel and Jayden were at track practice. The track is $\frac{2}{5}$ kilometers around.

- Angel ran 1 lap in 2 minutes.
- Jayden ran 3 laps in 5 minutes.

a. How many minutes does it take Angel to run one kilometer? What about Jayden?

b. How far does Angel run in one minute? What about Jayden?

c. Who is running faster? Explain your reasoning.

IM Commentary

Parts (a) and (b) of the task ask students to find the unit rates that one can compute in this context. Part (b) does not specify whether the units should be laps or km, so answers can be expressed using either one.

The purpose of part (c) is to give students an opportunity to make use of the unit rates that they found in parts (a) and (b). While it is possible for students to solve part (c) in other ways, the solution shown represents the kind of reasoning with unit rates that 7th graders should be able to do. It is important to note that the answer can be determined using different unit rates as long as the reasoning behind it is correct.

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Solution

a. We can create a table that shows how far each person runs for a certain number of laps:

Number of laps	Number of km
1	$\frac{2}{5}$
2	$\frac{4}{5}$
3	$\frac{6}{5}$

We can see from the table that 1 km is exactly half way between 2 and 3 laps. So it will take 2.5 laps to run 1 km.

Since it takes Angel 2 minutes to run 1 lap, she will take

$$\frac{2.5 \text{ laps}}{1 \text{ km}} \cdot \frac{2 \text{ minutes}}{1 \text{ lap}} = \frac{5 \text{ minutes}}{1 \text{ km}}.$$

So it takes Angel 5 minutes to run 1 km.

Since it takes Jayden 5 minutes to runs 3 laps, she runs 1 lap in $\frac{5}{3}$ minutes. Thus, it takes Jayden

$$\frac{2.5 \text{ laps}}{1 \text{ km}} \cdot \frac{5 \text{ minutes}}{3 \text{ laps}} = \frac{5}{2} \cdot \frac{5}{3} \text{ minutes/km} = \frac{25}{6} \text{ minutes/km} = 4\frac{1}{6} \text{ minutes/km.}$$

So it takes Jayden $4\frac{1}{6}$ minutes to run 1 km.

b. Angel runs 1 lap in 2 minutes so she runs $\frac{1}{2}$ lap in 1 minute. Since 1 lap is $\frac{2}{5}$ km, $\frac{1}{2}$ lap is $\frac{1}{5}$ km. So she also runs $\frac{1}{5}$ km in one minute.

Since Jayden runs 1 lap in $\frac{5}{3}$ minutes, she will run $\frac{3}{5}$ laps in 1 minute. Since Jayden runs 1 km in $\frac{25}{6}$ minutes, she will run $\frac{6}{25}$ km in 1 minute.

c. Jayden runs the same distance in less time than Angel (alternatively, Jayden runs farther in the same time than Angel), so Jayden is running faster than Angel.

Part, Whole, or Percent—Round 1 [KEY]

Directions: Find each missing value.

1.	1% of 100 is?	**1**	23.	10% of 22 is?	**2.2**
2.	2% of 100 is?	**2**	24.	20% of 22 is?	**4.4**
3.	3% of 100 is?	**3**	25.	30% of 22 is?	**6.6**
4.	4% of 100 is?	**4**	26.	50% of 22 is?	**11**
5.	5% of 100 is?	**5**	27.	25% of 22 is?	**5.5**
6.	9% of 100 is?	**9**	28.	75% of 22 is?	**16.5**
7.	10% of 100 is?	**10**	29.	80% of 22 is?	**17.6**
8.	10% of 200 is?	**20**	30.	85% of 22 is?	**18.7**
9.	10% of 300 is?	**30**	31.	90% of 22 is?	**19.8**
10.	10% of 500 is?	**50**	32.	95% of 22 is?	**20.9**
11.	10% of 550 is?	**55**	33.	5% of 22 is?	**1.1**
12.	10% of 570 is?	**57**	34.	15% of 80 is?	**12**
13.	10% of 470 is?	**47**	35.	15% of 60 is?	**9**
14.	10% of 170 is?	**17**	36.	15% of 40 is?	**6**
15.	10% of 70 is?	**7**	37.	30% of 40 is?	**12**
16.	10% of 40 is?	**4**	38.	30% of 70 is?	**21**
17.	10% of 20 is?	**2**	39.	30% of 60 is?	**18**
18.	10% of 25 is?	**2.5**	40.	45% of 80 is?	**36**
19.	10% of 35 is?	**3.5**	41.	45% of 120 is?	**54**
20.	10% of 36 is?	**3.6**	42.	120% of 40 is?	**48**
21.	10% of 37 is?	**3.7**	43.	120% of 50 is?	**60**
22.	10% of 37.5 is?	**3.75**	44.	120% of 55 is?	**66**

Part, Whole, or Percent—Round 2 [KEY]

Directions: Find each missing value.

1.	20% of 100 is?	20	23.	10% of 4 is?	0.4
2.	21% of 100 is?	21	24.	20% of 4 is?	0.8
3.	22% of 100 is?	22	25.	30% of 4 is?	1.2
4.	23% of 100 is?	23	26.	50% of 4 is?	2
5.	25% of 100 is?	25	27.	25% of 4 is?	1
6.	25% of 200 is?	50	28.	75% of 4 is?	3
7.	25% of 300 is?	75	29.	80% of 4 is?	3.2
8.	25% of 400 is?	100	30.	85% of 4 is?	3.4
9.	25% of 4000 is?	1000	31.	90% of 4 is?	3.6
10.	50% of 4000 is?	2000	32.	95% of 4 is?	3.8
11.	10% of 4000 is?	400	33.	5% of 4 is?	0.2
12.	10% of 4700 is?	470	34.	15% of 40 is?	6
13.	10% of 4600 is?	460	35.	15% of 30 is?	4.5
14.	10% of 4630 is?	463	36.	15% of 20 is?	3
15.	10% of 463 is?	46.3	37.	30% of 20 is?	6
16.	10% of 46.3 is?	4.63	38.	30% of 50 is?	15
17.	10% of 18 is?	1.8	39.	30% of 90 is?	27
18.	10% of 24 is?	2.4	40.	45% of 90 is?	40.5
19.	10% of 3.63 is?	0.363	41.	90% of 120 is?	108
20.	10% of 0.336 is?	0.0363	42.	125% of 40 is?	50
21.	10% of 37 is?	3.7	43.	125% of 50 is?	62.5
22.	10% of 37.5 is?	3.75	44.	120% of 60 is?	72

Fractional Percents—Round 1 [KEY]

Directions: Find the part that corresponds with each percent.

1.	1% of 100	**1**	23.	$\frac{1}{4}$% of 100	**$\frac{1}{4}$**
2.	1% of 200	**2**	24.	$\frac{1}{4}$% of 200	**$\frac{1}{2}$**
3.	1% of 400	**4**	25.	$\frac{1}{4}$% of 400	**1**
4.	1% of 800	**8**	26.	$\frac{1}{4}$% of 800	**2**
5.	1% of 1,600	**16**	27.	$\frac{1}{4}$% of 1,600	**4**
6.	1% of 3,200	**32**	28.	$\frac{1}{4}$% of 3,200	**8**
7.	1% of 5,000	**50**	29.	$\frac{1}{4}$% of 5,000	**$12\frac{1}{2}$**
8.	1% of 10,000	**100**	30.	$\frac{1}{4}$% of 10,000	**25**
9.	1% of 20,000	**200**	31.	$\frac{1}{4}$% of 20,000	**50**
10.	1% of 40,000	**400**	32.	$\frac{1}{4}$% of 40,000	**100**
11.	1% of 80,000	**800**	33.	$\frac{1}{4}$% of 80,000	**200**
12.	$\frac{1}{2}$% of 100	**$\frac{1}{2}$**	34.	1% of 1,000	**10**
13.	$\frac{1}{2}$% of 200	**1**	35.	$\frac{1}{2}$% of 1,000	**5**
14.	$\frac{1}{2}$% of 400	**2**	36.	$\frac{1}{4}$% of 1,000	**2.5**
15.	$\frac{1}{2}$% of 800	**4**	37.	1% of 4,000	**40**
16.	$\frac{1}{2}$% of 1,600	**8**	38.	$\frac{1}{2}$% of 4,000	**20**
17.	$\frac{1}{2}$% of 3,200	**16**	39.	$\frac{1}{4}$% of 4,000	**10**
18.	$\frac{1}{2}$% of 5,000	**25**	40.	1% of 2,000	**20**
19.	$\frac{1}{2}$% of 10,000	**50**	41.	$\frac{1}{2}$% of 2,000	**10**
20.	$\frac{1}{2}$% of 20,000	**100**	42.	$\frac{1}{4}$% of 2,000	**5**
21.	$\frac{1}{2}$% of 40,000	**200**	43.	$\frac{1}{2}$% of 6,000	**30**
22.	$\frac{1}{2}$% of 80,000	**400**	44.	$\frac{1}{4}$% of 6,000	**15**

Percent More or Less—Round 2 [KEY]

Directions: Find each missing value.

1.	100% of 20 is ___?	20
2.	10% of 20 is ___?	2
3.	10% more than 20 is ___?	22
4.	22 is ___ % more than 20?	10
5.	22 is ___% of 20?	110
6.	22 is 10% more than ___ ?	20
7.	110% of 20 is ___?	22
8.	10% less than 20 is ___?	18
9.	18 is ___% less than 20?	10
10.	18 is ___% of 20?	90
11.	18 is 10% less than ___?	20
12.	10% of 200 is ___?	20
13.	10% more than 200 is ___?	220
14.	220 is ___% of 200?	110
15.	220 is ___% more than 200?	10
16.	220 is 10% more than ___?	200
17.	110% of 200 is ___?	220
18.	10% less than 200 is ___?	180
19.	180 is ___% of 200?	90
20.	180 is ___% less than 200?	10
21.	180 is 10% less than ___?	200
22.	160 is ___% less than 200?	20

23.	15% of 60 is ___?	9
24.	15% more than 60 is ___?	69
25.	What is 115% of 60?	69
26.	69 is 115% of ___?	60
27.	69 is ___% more than 60?	15
28.	115% of 60 is ___?	69
29.	What is 15% less than 60?	51
30.	What % of 60 is 51?	85
31.	What % less than 60 is 51?	15
32.	What % less than 60 is 42?	30
33.	What % of 60 is 42?	70
34.	What is 20% more than 80?	96
35.	What is 30% more than 80?	104
36.	What is 140% of 80?	112
37.	What % of 80 is 104?	130
38.	What % more than 80 is 104?	30
39.	What % less than 80 is 56?	30
40.	What % of 80 is 56?	70
41.	1 is what % of 200?	$\frac{1}{2}$
42.	6 is what % of 200?	3
43.	24% of 200 is?	48
44.	24% more than 200 is?	248

Percent More or Less—Round 1 [KEY]

Directions: Find each missing value.

1.	100% of 10 is ___?	10	23.	15% of 80 is ___?	12
2.	10% of 10 is ___?	1	24.	15% more than 80 is ___?	92
3.	10% more than 10 is ___?	11	25.	What is 115% of 80?	92
4.	11 is ___% more than 10?	10	26.	92 is 115% of ___?	80
5.	11 is ___% of 10?	110	27.	92 is ___% more than 80?	15
6.	11 is 10% more than ___ ?	10	28.	115% of 80 is ___?	92
7.	110% of 10 is ___?	11	29.	What is 15% less than 80?	68
8.	10% less than 10 *is* ___?	9	30.	What % of 80 is 68?	85
9.	9 is ___% less than 10?	10	31.	What % less than 80 is 68?	15
10.	9 is ___% of 10?	90	32.	What % less than 80 is 56?	30
11.	9 is 10% less than ___?	10	33.	What % of 80 is 56?	70
12.	10% of 50 is ___?	5	34.	What is 20% more than 50?	60
13.	10% more than 50 is ___?	55	35.	What is 30% more than 50?	65
14.	55 is ___% of 50?	110	36.	What is 140% of 50?	70
15.	55 is ___% more than 50?	10	37.	What % of 50 is 85?	170
16.	55 is 10% more than ___?	50	38.	What % more than 50 is 85?	70
17.	110% of 50 is ___?	55	39.	What % less than 50 is 35?	30
18.	10% less than 50 is ___?	45	40.	What % of 50 is 35?	70
19.	45 is ___% of 50?	90	41.	1 is what % of 50?	2
20.	45 is ___% less than 50?	10	42.	6 is what % of 50?	12
21.	45 is 10% less than ___?	50	43.	24% of 50 is?	12
22.	40 is ___% less than 50?	20	44.	24% more than 50 is?	62

Fractional Percents—Round 2 [KEY]

Directions: Find the part that corresponds with each percent.

1.	10% of 30	**3**	23.	$10\frac{1}{2}$% of 100	**10.5**
2.	10% of 60	**6**	24.	$10\frac{1}{2}$% of 200	**21**
3.	10% of 90	**9**	25.	$10\frac{1}{2}$% of 400	**42**
4.	10% of 120	**12**	26.	$10\frac{1}{2}$% of 800	**84**
5.	10% of 150	**15**	27.	$10\frac{1}{2}$% of 1,600	**168**
6.	10% of 180	**18**	28.	$10\frac{1}{2}$% of 3,200	**336**
7.	10% of 210	**21**	29.	$10\frac{1}{2}$% of 6,400	**672**
8.	20% of 30	**6**	30.	$10\frac{1}{4}$% of 400	**41**
9.	20% of 60	**12**	31.	$10\frac{1}{4}$% of 800	**82**
10.	20% of 90	**18**	32.	$10\frac{1}{4}$% of 1,600	**164**
11.	20% of 120	**24**	33.	$10\frac{1}{4}$% of 3,200	**328**
12.	5% of 50	**2.5**	34.	10% of 1,000	**100**
13.	5% of 100	**5**	35.	$10\frac{1}{2}$% of 1,000	**105**
14.	5% of 200	**10**	36.	$10\frac{1}{4}$% of 1,000	**102.5**
15.	5% of 400	**20**	37.	10% of 2,000	**200**
16.	5% of 800	**40**	38.	$10\frac{1}{2}$% of 2,000	**210**
17.	5% of 1,600	**80**	39.	$10\frac{1}{4}$% of 2,000	**205**
18.	5% of 3,200	**160**	40.	10% of 4,000	**400**
19.	5% of 6,400	**320**	41.	$10\frac{1}{2}$% of 4,000	**420**
20.	5% of 600	**30**	42.	$10\frac{1}{4}$% of 4,000	**410**
21.	10% of 600	**60**	43.	10% of 5,000	**500**
22.	20% of 600	**120**	44.	$10\frac{1}{2}$% of 5,000	**525**