

Grade 8

Assignment Bundle

Name: _____ Class: _____

Invictus

By William Ernest Henley
1875

William Ernest Henley (1849-1903) was an English poet, critic, and editor. The following poem, published in 1875, is his best known work, which he wrote just after the amputation of his foot due to tuberculosis.¹ As you read, take notes on the poem's form and how it contributes to the tone of the speaker.

[1] Out of the night that covers me,
Black as the pit from pole to pole,
I thank whatever gods may be
For my unconquerable² soul.

[5] In the fell³ clutch of circumstance
I have not winced nor cried aloud.
Under the bludgeonings⁴ of chance
My head is bloody, but unbowed.

Beyond this place of wrath⁵ and tears
[10] Looms⁶ but the Horror of the shade,
And yet the menace⁷ of the years
Finds and shall find me unafraid.

It matters not how strait⁸ the gate,
How charged with punishments the scroll,
[15] I am the master of my fate,
I am the captain of my soul.



"I am the master of my fate, I am the captain of my soul" by Aristocrats-hat is licensed under CC BY-NC-ND 2.0.

Invictus by William Ernest Henley is in the public domain.

1. a disease caused by bacterial infection
2. **Unconquerable (adjective):** unable to be defeated
3. fierce, cruel, or terrible
4. beatings
5. **Wrath (noun):** violent anger
6. **Loom (verb):** to appear in a large, strange, or frightening form
7. **Menace (noun):** a threat
8. narrow (archaic)

Text-Dependent Questions

Directions: For the following questions, choose the best answer or respond in complete sentences.

1. Which of the following best describes a central theme of the text?
 - A. Identity is important to building self-confidence.
 - B. Independence means refusing to follow anyone else's rules or laws.
 - C. Resilience is the ability to keep going and to refuse to give up.
 - D. Sacrifice is necessary to make someone a hero.

2. How does the poem's use of darkness in its imagery contribute to the text's overall meaning?
 - A. The poem describes night covering the earth from "pole to pole" (line 2), suggesting that the speaker's outlook on the world is very bleak and hopeless.
 - B. The poem describes "night" (line 1) covering the speaker, symbolic of the adversity and/or suffering he faces.
 - C. The poem describes "the Horror of the shade" (line 10), suggesting that the speaker's hard times are only temporary.
 - D. The poem describes a dark "place of wrath and tears" (line 9), implying that the speaker is overcome by his depression.

3. PART A: Given the context of the poem, what does the title word "Invictus" most likely mean?
 - A. Careless
 - B. Lucky
 - C. Unfortunate
 - D. Undefeated

4. PART B: Which of the following quotes best supports the answer to Part A?
 - A. "I thank whatever gods may be / For my unconquerable soul." (Lines 3-4)
 - B. "In the fell clutch of circumstance /Under the bludgeonings of chance" (Lines 5-7)
 - C. "Beyond this place of wrath and tears / Looms but the Horror of the shade" (Lines 9-10)
 - D. "It matters not how strait the gate, / How charged with punishments the scroll" (Lines 13-14)

5. How does the poem's rhyme scheme and meter develop the tone?

Discussion Questions

Directions: *Brainstorm your answers to the following questions in the space provided. Be prepared to share your original ideas in a class discussion.*

1. How do you think the author would feel about receiving help from others? Cite the text in your answer.

2. In the context of this poem, how does a person overcome adversity? Cite evidence from this poem, your own experience, and other literature or art in your answer.

3. Do you think it is a good thing or a bad thing to rely on others in the face of adversity?

4. Can we ever truly be independent? Explain your answer.

5. There seems to be some tension in this text: the author claims to control his own life, yet he is subjected to the cruelty of chance and feels threatened by the future. Discuss this tension. According to the author, how can we be the “master of our fate” when there are circumstances beyond our control?

6. Do you believe that people can truly be “the master of [their] fate”? Is there really no such thing as destiny? Cite evidence from this text, your own experience, and other literature, art, or history in your answer.

Name: _____ Class: _____

Healing 'Brick City': A Newark Doctor Returns Home

By NPR Staff
2013

Newark, New Jersey is a notoriously high-crime city. Five of the city's last seven mayors have been indicted on criminal charges relating to political corruption. As of the 2006-2010 American Community Survey, 16% of Newark residents ages 25 and over had never attended high school. This is part of what makes the journey of Dr. Sampson Davis, an African-American man who grew up in the city and went on to become a physician and healthcare reform activist, so remarkable. As you read, take notes on Dr. Davis's explanation for his success and his view for the future of medicine.

- [1] When Sampson Davis was in high school, he and two of his friends made a pact that they would someday become doctors. All three of them did. Along with those friends — and now fellow doctors — George Jenkins and Rameck Hunt, Davis co-authored a 2003 book called *The Pact*, about that promise and the way it shaped their lives.

Now, in a new memoir, Davis describes his experience returning to the Beth Israel Hospital in Newark, N.J. — the hospital where he was born — as an emergency physician. Davis joins NPR's Scott Simon to talk about the book, *Living and Dying in Brick City: An E.R. Doctor Returns Home*.



"The Three Doctors" by The Three Doctors Foundation is in the public domain.

INTERVIEW HIGHLIGHTS

On returning to Newark after becoming a doctor

"My calling was a bit different. It was important for me to come back and become a beacon of hope, if you will, to show young people, especially, that education can change a life. It changed my life, and it saved my life in so many ways."

On encountering a childhood friend, Don "Snake" Moses, in the hospital

"He was a young guy that I grew up with in the streets of Newark. And my past wasn't always perfect. I grew up in a single-parent home with five siblings [in a] drug-infested community. And I always had hopes and aspirations¹ of doing more with my life, but I often say you can't aim for what you can't see. Growing up, I was surrounded by so much negative peer pressure and negativity, it wasn't long before I became a part of that fabric.

1. **Aspiration (noun):** a strong desire to achieve something high or great

- [5] “Snake and I was a part of a team that committed an armed robbery when I was 17 1/2. And I often say 17 1/2 because had I been 18, my story would have been written differently. But it was that life experience that changed me around. I was sentenced to two years’ probation. And I started back in high school and... earned straight A’s in high school, went off to college, and went down a different side of the fork in the road towards education. And Snake — Don Moses — stayed down the same road of crime, and he was in and out of jail.

“And it so happened that I finished college, I went off to medical school, came back home for my residency,² and the first day... I looked up at the board, and ‘Don Moses’ [was] written on the board in the trauma room. I’m like, ‘Wow, I know that name,’ and right below was written the word ‘deceased.’ So, I’m sitting in there, looking at the board and thinking, ‘What are the chances that this is the Don Moses that I know?’ And unfortunately, I sprinted down to the surgical ICU³ and his body was taken away, but his family was there. It was the Don Moses that I knew from childhood.”

On the need to acknowledge mental illness

“When you look at mental illness, in the inner city community particularly, it’s taboo.⁴ It’s almost like, ‘I can’t say to another person that I’m depressed, because it destroys, especially as a man talking to another man, it destroys the ‘man code,’ if you will. But in the book, I refer to a young man that I grew up with who was this bright, happy, young guy who I remember playing basketball with. I left, went off to college and medical school, and I returned. He didn’t look the same. He tried to act as if he was happy, but everything about him screamed depression. His depression stemmed⁵ from many things: He lost his mother, he lost his girlfriend — his fiancée. And he never [sought] help for it because it was one of those situations that I’ve come to understand, where you just don’t talk about it. He unfortunately took his life. He just spiraled out of control — he had no resources, and no ways of dealing with it or coping with it.”

On what it will take to improve health care in inner cities

“I think one is attention to the matter at hand and to realize that there is a need that exists in the cities as far as health care. Not only in the cities but in a great amount of rural⁶ areas, as well. I also feel that there has to be a program in place that encourage[s] youth from the beginning to become doctors, to become health care professionals. There has to be more programs that exist... to help the students matriculate⁷ through high school, through college, through medical school. Because more often, just like myself, you come back. You come back to home.”

2. “Residency” refers to the period of advanced training in a medical specialty that normally follows graduation from medical school.

3. Intensive Care Unit

4. **Taboo (adjective):** prohibited or restricted by social custom

5. **Stem (verb):** to come from

6. **Rural (adjective):** relating to the country, country people or life, or agriculture

7. to be enrolled at a college or university

On feeling the responsibility to give back

"Through my mother's way of handling life, she always made sure that I understood the need to give back.... She always said, 'Once you make it, you have to come back and help other people.' Too often, in Newark especially, I see so many professionals that do make it out — they don't return. And I think that's a crime in itself.... You have to have some social consciousness to give back, to be a part of making it better tomorrow."

©2013 National Public Radio, Inc. News report titled "Healing 'Brick City': A Newark Doctor Returns Home" was originally broadcast on NPR's Weekend Edition Saturday on February, 2013, and is used with the permission of NPR. Any unauthorized duplication is strictly prohibited.

Text-Dependent Questions

Directions: For the following questions, choose the best answer or respond in complete sentences.

1. PART A: Which of the following best identifies the central idea of this article?
 - A. Physicians have a responsibility to actively campaign for comprehensive healthcare and health insurance reform in the communities in which they work.
 - B. Davis overcame the influence of a tremendously negative environment to achieve great success and intends to use his training to better the community in which he grew up.
 - C. Dr. Sampson Davis and his two colleagues were able to become doctors due to a combination of hard work and luck; many Newark residents are not so fortunate.
 - D. Access to healthcare for people living in high-crime, high-density urban areas is the primary concern of physicians who live and work in these underserved communities.

2. PART B: Which phrase from the text best supports the answer to Part A?
 - A. "Growing up, I was surrounded by so much negative peer pressure and negativity, it wasn't long before I became a part of that fabric." (Paragraph 4)
 - B. "[I] committed an armed robbery when I was 17 1/2. And I often say 17 1/2 because had I been 18, my story would have been written differently." (Paragraph 5)
 - C. "There has to be more programs that exist... to help the students matriculate through high school, through college, through medical school." (Paragraph 8)
 - D. "And I think that's a crime in itself.... You have to have some social consciousness to give back, to be a part of making it better tomorrow." (Paragraph 9)

3. How does the following phrase contribute to the development of the main ideas of the text: "It was the Don Moses that I knew from childhood" (Paragraph 6)?
 - A. It illustrates how many people from underserved communities go down a negative path and it makes Davis's ascent all the more remarkable by comparison.
 - B. It demonstrates the impact the death of a childhood friend had on Davis, who would go on to become an emergency room physician.
 - C. It shows that a life of crime will inevitably result in an untimely death or permanent incarceration.
 - D. It advances the notion that Newark, New Jersey is an underserved city.

4. What is the author's main purpose in writing the article?
- A. To emphasize the extent to which growing up in a negative environment can stunt a young person's growth and development.
 - B. To show people how doctors from all different backgrounds can work together to make a difference in the quality of healthcare nationwide.
 - C. To inform and inspire people by sharing the narrative of a hard-working, compassionate person who prevailed over alarming circumstances to realize great success.
 - D. To encourage more young people who have endured difficult childhoods to pursue careers in emergency medicine.
5. How does Davis describe the relationship between growing up in Newark and returning to the city as a physician? Cite evidence from the text in your response.

Discussion Questions

Directions: *Brainstorm your answers to the following questions in the space provided. Be prepared to share your original ideas in a class discussion.*

1. What do you think Davis means when he says that “you can’t aim for what you can’t see” in paragraph 4? Do you agree? Can you think of another example where this might ring true, perhaps in your own life or in the world around you?

2. How does paragraph 7 shed light on how a rigid view of masculinity can negatively impact young men? Can you think of specific examples of ways in which society works to encourage men to repress their emotions instead of dealing with them?

3. Rudolf Virchow, a famous German physician and public health advocate, was famously quoted as saying that “physicians are the natural attorneys of the poor.” Do you agree? What responsibilities do doctors have to society and to the communities they serve?

4. Why do you think Davis considers it so important that individuals from a diverse variety of backgrounds are encouraged to pursue careers in medicine? Do you agree or disagree with his position?

5. In the context of this article, how does a person overcome adversity? Cite evidence from this text, your own experience, and other literature, art, or history in your answer.

Selecting Books for Your Child: Finding 'Just Right' Books

By: Kathleen Rogers

How can parents help their children find books that are not "too hard" and not "too easy" but instead are "just right"? Here's some advice.

Five finger rule

1. Choose a book that you think you will enjoy.
2. Read the second page.
3. Hold up a finger for each word you are not sure of, or do not know.
4. If there are five or more words you did not know, you should choose an easier book.
5. Still think it may not be too difficult? Use the five finger rule on two more pages.

Choose a book that is a good fit for you!

Read two or three pages and ask yourself these questions:

Will it be an easy, fun book to read?

- Do I understand what I am reading?
- Do I know almost every word?
- When I read it aloud, can I read it smoothly?
- Do I think the topic will interest me?

If most of your answers were "yes", this will be an easy book to read independently by yourself.

Will this book be too hard for me?

- Are there five or more words on a page that I don't know, or am unsure of?
- Is this book confusing and hard to understand by myself?
- When I read it aloud, does it sound choppy and slow?

If most of your answers were "yes," this book is too hard. You should wait awhile before you read this book. Give the book another try later, or ask an adult to read the book to you.

Tips on reading with your child

- When they can't read the word, say...
- Can you sound it out?
- Fingertap it.
- Can you think of the word or movement that helps you remember that vowel sound?
- What is the first and last sound? What word would make sense?
- Does it have a pattern that you have seen in other words? (ex-an, ack)
- How does the word begin?
- You said _____. Does that make sense?
- What word would make sense that would start with these sounds?
- Put your finger under the word as you say it.

When they want to read a book that is too hard, say...

- Let's read it together.
- This is a book you will enjoy more if you save it until you are older — or later in the year.
- [Be honest!] When people read books that are too hard for them, they often skip important parts. You will have more fun with this book if you wait until you can read it easily.

HERE'S THE IMPACT OF READING 20 MINUTES PER DAY!

A student who reads

20:00

minutes per day

will be exposed to
1.8 MILLION
words per year
and scores in
90th PERCENTILE
on standardized tests

A student who reads

5:00

minutes per day

will be exposed to
282,000
words per year
and scores in
50th PERCENTILE
on standardized tests

A student who reads

1:00

minute per day

will be exposed to
8,000
words per year
and scores in
10th PERCENTILE
on standardized tests

Source: Nagy, Anderson and Herman, 1987

Home Reading Log

Student Information	
---------------------	--

Student Name		Grade Level	
School Name		Teacher	

Log

[illegible]

Applying Properties for Powers with the Same Base

► Rewrite each expression as a single power.

1 $6^4 \cdot 6^4$

2 $(-5^5)^2$

3 $\frac{2^9}{2^5}$

4 $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3^2$

5 $\frac{12^5 \cdot 12^7}{-12^4}$

6 $\left(\frac{7^5}{7^2}\right)^2$

► Evaluate each expression.

7 $\frac{4^8}{4^5}$

8 $(-10) \cdot (-10)^4$

9 $\left(\frac{(-3)^4}{(-3)^2}\right)^3$

► What value of x makes the equation true?

10 $\frac{8^x}{8^5} = 8^7$

11 $(-11)^x \cdot (-11)^4 = \frac{(-11)^{10}}{(-11)^3}$

12 $(6^x)^{10} = \frac{(6^{12})^2}{6^4}$

13 Explain how you solved for x in problem 12.

Applying Properties for Powers with the Same Exponent

► Rewrite each expression as a single power.

1 $9^4 \cdot 10^4$

2 $(12 \cdot 6)^3$

3 $\frac{3^3}{2^3}$

4 $\frac{6^2}{2^2}$

5 $(-5)^6 \cdot (-7)^6$

6 $\left(\frac{6^4}{12^4}\right)^2$

► Rewrite each expression as a product of two powers or quotient of two powers.

7 $5^5(16^2 \cdot 5^3)^3$

8 $\left(\frac{8^4 \cdot 5^3}{8^5}\right)^2$

9 $\left(\frac{5^8 \cdot 3^7}{5^4}\right)^{10}$

- 10 How does multiplying powers with the same base differ from multiplying powers with the same exponent but different bases?

Applying Properties of Negative Exponents

- Rewrite each expression using only positive exponents. The answers are mixed up at the bottom of the page. Cross out the answers as you complete the problems.

1 $7^3 \cdot 16^{-9}$

2 $\frac{8^{-6}}{21^{-4}}$

3 $\left(\frac{7}{16}\right)^{-3}$

4 $16^3 \cdot (-7)^{-3}$

5 $(8 \cdot 21)^{-4}$

6 $8 \cdot 21^{-3}$

7 $\frac{11^{-7} \cdot 5^9}{6^9}$

8 $\frac{11^{-7} \cdot 5^9}{6^{-9}}$

9 $6^9 \cdot 11^{-7} \cdot 5^{-9}$

10 $\frac{3^5 \cdot (-4)^{-10}}{7^9 \cdot 21^{-4}}$

11 $\frac{(-21)^{-4} \cdot (-4)^0}{3^{-5} \cdot 7^{-9}}$

12 $\left(\frac{3}{7}\right)^{-5} \cdot (-21)^{-4} \cdot (-4)^2$

Answers

$$\frac{1}{(8 \cdot 21)^4}$$

$$\frac{6^9}{11^7 \cdot 5^9}$$

$$\frac{16^3}{7^3}$$

$$\frac{7^5 \cdot (-4)^2}{3^5 \cdot (-21)^4}$$

$$\frac{21^4}{8^6}$$

$$\frac{6^9 \cdot 5^9}{11^7}$$

$$\frac{16^3}{(-7)^3}$$

$$\frac{3^5 \cdot 21^4}{7^9 \cdot (-4)^{10}}$$

$$\frac{3^5 \cdot 7^2}{(-21)^4}$$

$$\frac{8}{21^3}$$

$$\frac{5^9}{11^7 \cdot 6^9}$$

$$\frac{7^3}{16^9}$$

Applying Properties of Integer Exponents

► Evaluate each expression.

1 $18^{-4} \cdot 6^7$

2 $3^4 \cdot 3^{-6} \cdot 9^0$

3 $\left(\frac{3^{-4} \cdot 3^6}{6^3 \cdot 6^{-1}}\right)^{-2}$

► Write each expression using only positive exponents.

4 $19^{-3} \cdot 19 \cdot 19^{-4} \cdot 19^3$

5 $\frac{6^{-3} \cdot 17^3 \cdot 2}{6^5 \cdot 17^{-4} \cdot 2^{-1}}$

6 $24^{-3} \cdot 24^7 \cdot (24^{-3})^4 \cdot 24^9$

7 $\left(\frac{7^{-3} \cdot 3^{-8}}{7^{-2} \cdot 3^{-2}}\right)^{-4}$

8 $(2^{-1} \cdot 3^0)^{-3} \cdot (2^0 \cdot 5^3)^5$

9 $\left(\frac{5^6 \cdot 3^{-3}}{3^{-3}}\right)^4$

10 How could you have simplified problem 7 in a different way?

Applying Properties of Exponents to Generate Equivalent Expressions I—Round 1 [KEY]

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. All letters denote numbers.

1.	$2^2 \cdot 2^3$	2^5
2.	$2^2 \cdot 2^4$	2^6
3.	$2^2 \cdot 2^5$	2^7
4.	$3^7 \cdot 3^1$	3^8
5.	$3^8 \cdot 3^1$	3^9
6.	$3^9 \cdot 3^1$	3^{10}
7.	$7^6 \cdot 7^2$	7^8
8.	$7^6 \cdot 7^3$	7^9
9.	$7^6 \cdot 7^4$	7^{10}
10.	$11^{15} \cdot 11$	11^{16}
11.	$11^{16} \cdot 11$	11^{17}
12.	$2^{12} \cdot 2^2$	2^{14}
13.	$2^{12} \cdot 2^4$	2^{16}
14.	$2^{12} \cdot 2^6$	2^{18}
15.	$99^5 \cdot 99^2$	99^7
16.	$99^6 \cdot 99^3$	99^9
17.	$99^7 \cdot 99^4$	99^{11}
18.	$5^8 \cdot 5^2$	5^{10}
19.	$6^8 \cdot 6^2$	6^{10}
20.	$7^8 \cdot 7^2$	7^{10}
21.	$r^8 \cdot r^2$	r^{10}
22.	$s^8 \cdot s^2$	s^{10}

23.	$6^3 \cdot 6^2$	6^5
24.	$6^2 \cdot 6^3$	6^5
25.	$(-8)^3 \cdot (-8)^7$	$(-8)^{10}$
26.	$(-8)^7 \cdot (-8)^3$	$(-8)^{10}$
27.	$(0.2)^3 \cdot (0.2)^7$	$(0.2)^{10}$
28.	$(0.2)^7 \cdot (0.2)^3$	$(0.2)^{10}$
29.	$(-2)^{12} \cdot (-2)^1$	$(-2)^{13}$
30.	$(-2.7)^{12} \cdot (-2.7)^1$	$(-2.7)^{13}$
31.	$1.1^6 \cdot 1.1^9$	1.1^{15}
32.	$57^6 \cdot 57^9$	57^{15}
33.	$x^6 \cdot x^9$	x^{15}
34.	$2^7 \cdot 4$	2^9
35.	$2^7 \cdot 4^2$	2^{11}
36.	$2^7 \cdot 16$	2^{11}
37.	$16 \cdot 4^3$	4^5
38.	$3^2 \cdot 9$	3^4
39.	$3^2 \cdot 27$	3^5
40.	$3^2 \cdot 81$	3^6
41.	$5^4 \cdot 25$	5^6
42.	$5^4 \cdot 125$	5^7
43.	$8 \cdot 2^9$	2^{12}
44.	$16 \cdot 2^9$	2^{13}

Applying Properties of Exponents to Generate Equivalent Expressions I—Round 2 [KEY]

Directions: Simplify each expression using the laws of exponents. Use the least number of bases possible and only positive exponents. All letters denote numbers.

1.	$5^2 \cdot 5^3$	5^5
2.	$5^2 \cdot 5^4$	5^6
3.	$5^2 \cdot 5^5$	5^7
4.	$2^7 \cdot 2^1$	2^8
5.	$2^8 \cdot 2^1$	2^9
6.	$2^9 \cdot 2^1$	2^{10}
7.	$3^6 \cdot 3^2$	3^8
8.	$3^6 \cdot 3^3$	3^9
9.	$3^6 \cdot 3$	3^{10}
10.	$7^{15} \cdot 7$	7^{16}
11.	$7^{16} \cdot 7$	7^{17}
12.	$11^{12} \cdot 11^2$	11^{14}
13.	$11^{12} \cdot 11^4$	11^{16}
14.	$11^{12} \cdot 11^6$	11^{18}
15.	$23^5 \cdot 23^2$	23^7
16.	$23^6 \cdot 23^3$	23^9
17.	$23^7 \cdot 23^4$	23^{11}
18.	$13^7 \cdot 13^3$	13^{10}
19.	$15^7 \cdot 15^3$	15^{10}
20.	$17^7 \cdot 17^3$	17^{10}
21.	$x^7 \cdot x^3$	x^{10}
22.	$y^7 \cdot y^3$	y^{10}

23.	$7^3 \cdot 7^2$	7^5
24.	$7^2 \cdot 7^3$	7^5
25.	$(-4)^3 \cdot (-4)^{11}$	$(-4)^{14}$
26.	$(-4)^{11} \cdot (-4)^3$	$(-4)^{14}$
27.	$(0.2)^3 \cdot (0.2)^{11}$	$(0.2)^{14}$
28.	$(0.2)^{11} \cdot (0.2)^3$	$(0.2)^{14}$
29.	$(-2)^9 \cdot (-2)^5$	$(-2)^{14}$
30.	$(-2.7)^5 \cdot (-2.7)^9$	$(-2.7)^{14}$
31.	$3.1^6 \cdot 3.1^6$	3.1^{12}
32.	$57^6 \cdot 57^6$	57^{12}
33.	$z^6 \cdot z^6$	z^{12}
34.	$4 \cdot 2^9$	2^{11}
35.	$4^2 \cdot 2^9$	2^{13}
36.	$16 \cdot 2^9$	2^{13}
37.	$16 \cdot 4^3$	4^5
38.	$9 \cdot 3^5$	3^7
39.	$3^5 \cdot 9$	3^7
40.	$3^5 \cdot 27$	3^8
41.	$5^7 \cdot 25$	5^9
42.	$5^7 \cdot 125$	5^{10}
43.	$2^{11} \cdot 4$	2^{13}
44.	$2^{11} \cdot 16$	2^{15}



Extending the Definitions of Exponents, Variation 1

Tags: **MP 8**

Alignments to Content Standards: **8.EE.A.1**

Student View

Task

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

a.

The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2				

b.

If you know the size of the population at a certain time, how do you find the population one hour later?

c.



d.

Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study? What about 3 hours before?

e.

If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

f.

What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

g.

Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

h.

Use the context to explain why it makes sense that $2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$. That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by $\frac{1}{2}$.

IM Commentary

This is an instructional task meant to generate a conversation around the meaning of negative integer exponents. While it may be unfamiliar to some students, it is good for them to learn the convention that negative time is simply any time before $t = 0$.

Students will struggle to put their explanation for part (h) together. A teacher might want to have the students do parts (a) - (g) as a precursor to providing an explanation like the one given in the solution for part (h).

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task leads students systematically through the use of repeated reasoning to understand algorithms and make generalizations about patterns. (MP.8) This task could be used as an



about the results each time. After the students have worked through g , all results of $a \cdot g$ are posted for students to see; the teacher could have students individually write an explanation for h . Students could then have small and/or whole group discussion to finalize an answer with the teacher guiding the discussion to the correct answer and explicitly pointing out, if needed, the repeated reasoning being used and the generalization of the pattern that is developed. The core of this task – generalizing to negative exponents – is also about making use of structure (MP. 7). Students see the structure of adding 1 to the exponent corresponding to multiplying by 2. Then they see that this works for negative exponents as well.

Solution

a.

What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2	4	8	16	32

b.

You multiply it by 2, since it doubled.

c.

The values predicted by Seth's equation agree exactly with those in the table above; Seth's equation works because it predicts a doubling of the population every hour. Marco's doesn't because it doesn't double the new population you have – instead it is doubling the time. Marco's equation predicts a linear growth of only two thousand bacteria per hour.

d.

Since the population is multiplied by 2 every hour we would have to divide by 2 (which is the same as multiplying by $\frac{1}{2}$) to work backwards. The population 1 hour before the study started would be

$$\frac{1}{2} \cdot 2 = 1 \text{ thousand,}$$

and the population 3 hours before the study started would be

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = 0.25 \text{ thousand,}$$

so 250 bacteria.

e.



f.

Time before the study started would be negative time; for example one hour before the study began was $t = -1$.

Hours into study	-3	-2	-1	0	1	2	3	4
Population (thousands)	$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$	$\frac{1}{2} \cdot 1 = 0.5$	1	2	4	8	16	32

g.

Since one hour before the study started would be $t = -1$, we would simply plug this value into Seth's equation:

$$2 \cdot (2)^{-1} = 2 \cdot \left(\frac{1}{2}\right) = 1 \text{ thousand.}$$

Three hours before would be $t = -3$. Using the equation:

$$2 \cdot (2)^{-3} = \frac{2}{2^3} = 0.25 \text{ thousand,}$$

giving us the same answers as we got through reasoning.

h.

Since the bacteria double every hour, we multiply the population by two for every hour we go forward in time. So if we want to know what the population will be 8 hours after the experiment started, we need to multiply the population at the start ($t = 0$) by 2 eight times. This explains why we raise 2 to the number of hours that have passed to find the new population; repeatedly doubling the population means we repeatedly multiply the population at $t = 0$ by 2.

In this context, negative time corresponds to time *before* the experiment started. To figure out what the population was before the experiment started we have to “undouble” (or multiply by $\frac{1}{2}$) for every hour we have to go back in time. So if we want to know what the population was 8 hours before the experiment started, we need to multiply the population at the start ($t = 0$) by $\frac{1}{2}$ eight times. The equation indicates that we should raise 2 to a power that corresponds to the number of hours we need to go back in time. For every hour we go back in time, we multiply by $\frac{1}{2}$. So it makes sense in this context that raising 2 to the -8 power (or any negative integer power) is the same thing as repeatedly multiplying $\frac{1}{2}$ 8 times (or the opposite of the power you raised 2 to). In other words, it makes sense in this context that

$$2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}.$$



Typeset May 4, 2016 at
18:58:52. Licensed by
Illustrative Mathematics

under a
Creative Commons Attribution-
NonCommercial-ShareAlike 4.0
International License.